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La revista Científica, como su nombre lo indica, es un medio de difusión del conocimiento científico y tecnológico, el cual se caracteriza por publicar artículos científicos originales que contribuyen al conocimiento en diversas áreas de la ciencia y la tecnología, así como artículos donde se presentan una revisión profunda sobre diversos temas de interés actual. Continuando con su característica multidisciplinaria en este número, la revista Científica publica artículos relacionados con la ingeniería mecánica, la ingeniería eléctrica y la electrónica.

El primer artículo presenta la validación con datos experimentales obtenidos en el aire al nivel del mar de diversos modelos para el cálculo de campo reducido \( (E/N) \) en descargas eléctricas. Aquí se muestra que un parámetro importante que gobierna el comportamiento de la descarga es la relación del campo eléctrico \( E \) a la presión \( p \) del gas o más exactamente a la densidad de moléculas \( N \) del gas.

El segundo trabajo presenta un estudio de la influencia de cuatro de los principales parámetros que intervienen en la eficiencia de separación de un ciclón con entrada tangencial. Aquí se consideran los efectos de algunas propiedades del gas y de las partículas por separar, así como los efectos de algunas dimensiones geométricas de los ciclones.

El siguiente artículo examina tres métodos para diagnóstico paramétrico de turbinas de gas, en el cual el funcionamiento de los diferentes métodos se simula en condiciones idénticas al desarrollo de fallas y errores aleatorios de medición. Los objetivos de esta investigación son afinar los métodos, compararlos y escoger el mejor, con base en criterios probabilísticos para el reconocimiento correcto e incorrecto de las clases de fallas.

En «VSLI Fuzzy Cells» se presenta el desarrollo de celdas básicas para la construcción de funciones de membresía trapezoidales, las cuales se encuentran formadas por un circuito de sustracción de corriente, un multiplicador-divisor y circuitos de forma \( S-Z \) usando tecnología CMOS de 0.18 \( \mu \text{m} \) en modo de corriente.

Finalmente se presenta el uso de técnicas modernas de elementos finitos para obtener eficientemente la respuesta a la frecuencia de máquinas síncronas en reposo, esto responde al continúo interés que existe por evitar pruebas experimentales en máquinas de alta potencia, debido a que implican altos costos y riesgo de daños. El modelo de elementos finitos desarrollado en este trabajo también toma en consideración los circuitos externos conectados a la máquina a través de una solución simultánea de las ecuaciones de los circuitos y del dominio electromagnético.

Así, de esta manera, el presente número publica contribuciones que colaboran e incrementan el conocimiento en campos relativos las ingenierías eléctrica, mecánica y electrónica.
Cálculo del campo reducido \((E/N)\) en descargas eléctricas

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1. Resumen

Los diferentes modelos propuestos del líder se han validado con datos experimentales obtenidos en aire al nivel del mar. Un parámetro importante que gobierna el comportamiento de la descarga es la relación del campo eléctrico \(E\) a la presión \(p\) del gas o más exactamente a la densidad de moléculas \(N\) del gas. En este artículo se presentan los valores del campo reducido \((E/N)\) y del gradiente de potencial \((E)\) utilizándo el modelo de Gallinberti con datos experimentales obtenidos a 2930 m.s.n.m. \((\delta = 0.7)\).

Palabras clave: descargas eléctricas, canal del líder, campo reducido, gradiente de potencial.

2. Abstract (Calculus of Reduced Field \((E/N)\) in Electrical Discharges)

The different models proposed of the channel leader have been validated with experimental data obtained in atmospheric air at sea level. An important parameter that governs the behavior of the electrical discharge is the relation of the electric field \(E\) to the pressure \(p\) of the gas or more exactly to molecules density \(N\) of the gas. In this paper values of the reduced field \((E/N)\) and potential gradient \((E)\) obtained using the model of Gallinberti with experimental data with air at 2 240 meters at sea level are reported.

3. Introduction

La principal diferencia entre los modelos propuestos para el canal del líder, estaría en si consideran o no un equilibrio local termodinámico [1,2,3]. El modelo de Goldinberti, vigente hasta ahora, no lo supone, puede ser resuelto analíticamente y está basado en las conclusiones de Braginskii, que consideran al canal del líder como un núcleo cilíndrico homogéneo formado de un plasma débilmente ionizado, donde debido a las colisiones entre los electrones y las moléculas se transfiere energía del campo eléctrico al gas, la evolución en el tiempo de las condiciones internas de una sección del canal puede ser estudiada sobre la base de procesos termodinámicos e hidrodinámicos, los cuales son generados por la energía de entrada.

De acuerdo con Braginskii [3], el canal del líder puede ser considerado como un núcleo cilíndrico homogéneo envuelto por una nube densa de gas, con presión, temperatura y densidad de partículas constantes sobre la sección transversal del núcleo; la variación de estos parámetros con respecto a los valores del gas circundante no alterado se concentra entonces en la nube. Para un segmento del canal de longitud unitaria, la ecuación de conservación de energía puede ser escrita separadamente para el núcleo y la nube como:

\[
\frac{dW_i}{dt} + \rho \frac{d(\varepsilon \rho)}{dt} = Q_i
\]

(1)

\[
\left( \varepsilon + \frac{p}{mN} \right) \frac{dM}{dt} = Q_r + Q_T
\]

(2)

donde:

- \(W_i\) = energía interna
- \(Q_r\) = pérdidas por conducción térmica
- \(Q_T\) = pérdidas por radiación
- \(Q\) = potencia térmica de entrada
- \(m\) = masa total
- \(\varepsilon\) = energía interna por unidad de masa
- \(m^*\) = densidad de moléculas del gas en el canal del núcleo
- \(p\) = presión en el canal del núcleo
- \(m\) = masa de las moléculas del gas
- \(M = mN\pi a^2\)
- \(W_i = Mc = mN\pi a^2\varepsilon\)
- \(a\) = radio del núcleo

La presión en el núcleo se toma igual al valor atmosférico \(p\), y se supone que el gas en el canal del líder se comporta como un gas perfecto y que el grado de ionización, definido como la relación de iones positivos a la densidad de moléculas del gas \((\alpha/N)\), es
mucho más bajo que la unidad, por lo cual se tiene que $p = NKT$, con $T$ = temperatura en grados Kelvin y $K$ = constante de Boltzmann.

Como no se considera equilibrio termodinámico, la energía interna por unidad de masa se puede calcular con buena aproximación como:

$$\varepsilon^* = \frac{1}{m} \frac{KT}{\gamma - 1} = \frac{l}{\gamma - 1} \frac{p}{mN}$$  \hspace{1cm} (3)

donde:

$\gamma = \text{relación entre calores específicos a volumen y presión constantes}$

La potencia de entrada $Q_e$ puede estimarse como la potencia eléctrica por unidad de longitud $EI$, $Q_e$ y $Q_e$ pueden despreciables si la temperatura del canal no excede $10^5$ K. Considerando la corriente $I$ el núcleo como una variable independiente externa y que las concentraciones de iones positivos y de electrones son las mismas, se tiene para el núcleo que:

$$\frac{dN_e}{dt} = v_i N_e - a_i N_e n_e^+$$  \hspace{1cm} (4)

donde:

$v_i$ = frecuencia de ionización

$a_i$ = coeficiente de recombinación

$N_e$ = número de electrones por unidad de longitud en el canal

$n_e^+$ = densidad de iones positivos o electrones

Si el canal se comporta como un conductor resistivo el campo eléctrico puede ser expresado por

$$E = \frac{I}{N_e \varepsilon \mu_e}$$  \hspace{1cm} (5)

donde:

$\mu_e$ = movilidad electrónica

$\varepsilon$ = carga del electrón

$E$ = gradiente de potencial del líder

**Conductividad del líder**

En función de las pérdidas despreciables en la frontera, el canal se expande como una masa constante, $m_{te} = N_e$, donde $a_y$ y $N_e$ son el radio y número de moléculas del gas a un tiempo $t_y$. Se ha observado que para el valor $E/N$ y $n_e^+$, los cuales pueden ser determinados para el canal, las constantes de tiempo de ionización $1/n_e^+$ y de recombinación $1/(a_i n_e^+)$ son mucho menores que $1/\mu_e$; el cual a su vez es mucho menor que el tiempo de desarrollo del líder. En consecuencia las densidades de iones positivos y electrones corresponden a un equilibrio entre los procesos de ionización y de recombinación, por lo que

$$n_e^+ = \frac{v_i}{a_i}$$  \hspace{1cm} (6)

y la ecuación (5) puede entonces escribirse como:

$$E = \frac{I}{N_{te} \varepsilon \mu_e} = \frac{1}{N_{te} \varepsilon \mu_e} \frac{v_i}{a_i} N_{te}$$  \hspace{1cm} (7)

Como $V/a$, $\nu_i N_{te}$ y $N_{te}$ son solamente funciones del campo reducido $E/N$ mientras que $N_{te} \varepsilon \mu_e$ es una constante, la ecuación (7) puede usarse para calcular $E/N$ como función únicamente de la corriente del líder. Esto implica que el grado de ionización y la conductividad del líder:

$$i = \frac{n_e^+}{N_{te}} = \frac{1}{N_{te}} \frac{v_i}{a_i} N_{te}$$  \hspace{1cm} (8)

$$\sigma = \frac{1}{N_{te} \varepsilon \mu_e} = \frac{1}{N_{te} \varepsilon \mu_e} \frac{v_i}{a_i} N_{te}$$  \hspace{1cm} (9)

únicamente son funciones de la corriente del líder.

De las ecuaciones anteriores se puede observar que la corriente $I$ y la conductividad determinan el campo reducido $E/N$, el cual a su vez determina el grado de ionización $i$ y la conductividad $\sigma$. Ocurre una retroalimentación en lazo cerrado típica con una alta ganancia fijada por las características diferenciales de la función de transferencia de retroalimentación $(v_i/a_i) \mu_e$; la salida $E/N$ por tanto se estabiliza, independientemente de las fluctuaciones de la corriente de entrada.

**Expansión del canal del líder**

Combinando las ecuaciones (1), (2), (3) y (4) considerando $p = p_o$ y $\gamma = constante$, se encuentra la siguiente ecuación para la expansión del líder:
La potencia de entrada puede ser expresada en la forma:

$$EI = \frac{E}{N} I N_o \pi a_o^2$$

(11)

donde $E/N$ es casi constante. La ecuación (11), para la expansión del líder, puede ser integrada directamente para dar la sección transversal del canal a un tiempo $t$ y una posición $x$:

$$\left(\frac{\pi a^2}{4}\right)_i = \sqrt{\left(\frac{\pi a^2}{4}\right)_0} + 2 \frac{\sqrt{\pi a_o^2}}{\gamma N_p} \int_0^t dt$$

(12)

con

$t_o$ = tiempo al cual la cabeza del líder cruza la posición $x$ y se forma la sección del canal.

Se ha observado experimentalmente que la carga $q$ por unidad de longitud es constante durante la propagación del líder a pesar de las fluctuaciones de corriente, por lo cual la ecuación (12) puede reescribirse como:

$$\int_0^t dt = q \int_0^t (x_t - x)$$

(13)

con

$x =$ coordenada curvilínea a lo largo de la trayectoria real del líder

$x_t =$ longitud real del líder al tiempo $t$.

### Cuadro 1. Valores calculados de $E/N$.  

<table>
<thead>
<tr>
<th>$I$ (A)</th>
<th>TOPILEJO $E/N$(V/cm²)</th>
<th>NIVEL DEL MAR $E/N$(V/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>$8.91 \times 10^{-16}$</td>
<td>$8.53 \times 10^{-16}$</td>
</tr>
<tr>
<td>0.5</td>
<td>$9.50 \times 10^{-16}$</td>
<td>$9.40 \times 10^{-16}$</td>
</tr>
<tr>
<td>1.0</td>
<td>$9.83 \times 10^{-16}$</td>
<td>$9.74 \times 10^{-16}$</td>
</tr>
<tr>
<td>2.0</td>
<td>$9.96 \times 10^{-16}$</td>
<td>$9.91 \times 10^{-16}$</td>
</tr>
<tr>
<td>5.0</td>
<td>$1.00 \times 10^{-15}$</td>
<td>$1.00 \times 10^{-15}$</td>
</tr>
<tr>
<td>10.0</td>
<td>$1.00 \times 10^{-15}$</td>
<td>$1.00 \times 10^{-15}$</td>
</tr>
<tr>
<td>20.0</td>
<td>$1.00 \times 10^{-15}$</td>
<td>$1.00 \times 10^{-15}$</td>
</tr>
<tr>
<td>50.0</td>
<td>$1.00 \times 10^{-15}$</td>
<td>$1.00 \times 10^{-15}$</td>
</tr>
</tbody>
</table>

### Cuadro 2. Valores calculados de $E$.  

<table>
<thead>
<tr>
<th>$I$ (A)</th>
<th>TOPILEJO $E$(MV/m)</th>
<th>NIVEL DEL MAR $E$(MV/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.201</td>
<td>0.267</td>
</tr>
<tr>
<td>1.6</td>
<td>0.149</td>
<td>0.198</td>
</tr>
<tr>
<td>1.0</td>
<td>0.118</td>
<td>0.156</td>
</tr>
<tr>
<td>2.0</td>
<td>0.084</td>
<td>0.112</td>
</tr>
<tr>
<td>3.0</td>
<td>0.069</td>
<td>0.092</td>
</tr>
<tr>
<td>4.0</td>
<td>0.060</td>
<td>0.080</td>
</tr>
</tbody>
</table>

4. **Body**

4.1 **Cálculo y análisis de $E/N$ y $E$**

Investigaciones realizadas por el Laboratorio de Pruebas de Equipos y Materiales de la Comisión Federal de Electricidad (LAPEM-CFE) en Topilejo, DF, a 2930 m.s.n.m. [4], indican que la carga asociada por unidad de longitud y velocidad de propagación del líder, no parecen ser influenciadas significativamente por la densidad del aire ($p = 728$ mbars, $\delta = 0.7$, $v = 1.5$ cm/s). Considerando $a_o = 1$ mm y $T_o = 1000$ K, en el cuadro 1 se reportan y en la figura 1 se grafican los valores calculados de $E/N$ en función de la corriente, para Topilejo y a nivel del mar ($p = 1.013$ mbars).

Similarmente en el cuadro 2 y en la figura 2, se muestran respectivamente los valores calculados y las gráficas de $E$ en función de la longitud del líder (ecuaciones (12) y (13)) ($a_o = 1$ mm, $T_o = 1000$ K, $q = 35$ µC, $E/N = 8.91 \times 10^{-16}$ V/cm² = etc).
Se observa de la figura 1 que el campo reducido $E/N$ no es significativamente afectado por la densidad del aire.

El cálculo del gradiente de potencial del líder en función de su longitud, empleando el modelo de Gallimberti, es, de acuerdo con la figura 2, mayor a nivel del mar que a una altitud de 2 930 m.

5. Conclusiones

Investigaciones previas demostraron que la carga asociada por unidad de longitud y velocidad de propagación del líder, no son influenciadas significativamente por la densidad del aire; con el presente trabajo también es posible afirmar que, similarmente, el campo reducido $E/N$ no es afectado de manera importante por la densidad del aire ni por variaciones del diámetro y temperatura del canal del líder.

El gradiente de potencial del líder en función de su longitud es 25% mayor a nivel del mar que a una altura de 2 930 m.s.n.m.

6. Referencias


Estudio de los parámetros que afectan la eficiencia de separación de los separadores tipo ciclón

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1. Resumen

En este trabajo se presenta un estudio de la influencia de cuatro parámetros que intervienen en la eficiencia de separación de un ciclón con entrada tangencial. Se consideran los efectos de algunas propiedades del gas y de las partículas a separar, así como los efectos de algunas dimensiones geométricas de los ciclones. El estudio paramétrico se lleva a cabo utilizando un modelo matemático propuesto anteriormente por los autores, previamente comparado con resultados experimentales mostrando un buen desempeño en la predicción de la eficiencia de separación de estos equipos.

2. Abstract (Study of the Parameters Affecting Cyclone Separation Efficiency)

A study of the influence of four affecting parameters on the cyclone separation efficiency is presented in this work. Some gas and particle properties, as well as some geometric dimensions are taken into account. The parametric study was carried out using a mathematical model previously proposed by the authors. This model was compared with experimental results, showing a good agreement between separation efficiency predictions and experiments.

3. Introducción

Los separadores tipo ciclón, también conocidos simplemente como «ciclones» son probablemente los equipos más usados en el mundo para capturar partículas [1].

Los ciclones son mayormente usados para remover polvo o partículas sólidas del aire u otros gases. Estos equipos son principalmente utilizados en la industria en procesos de separación de sólidos de corrientes de gas.

Las principales ventajas de los ciclones son: bajo costo de operación (al no poseer partes móviles, los costos de mantenimiento son mínimos), simplicidad en su construcción y capacidad de operar a altas temperaturas y presiones [2].

Tradicionalmente, los ciclones se han usado en la industria para capturar partículas relativamente grandes ($d_p > 10 \mu m$) [3]. Sin embargo, estos equipos también pueden usarse para capturar partículas con diámetros de 10, 2.5 e incluso 1 \mu m, modificando sus dimensiones. Esto permite usarlos en tareas de monitoreo ambiental de partículas [4].

La amplia utilización de estos equipos crea la necesidad de contar con modelos matemáticos para evaluar su operación así como de contar con metodologías simples y efectivas para su diseño basadas en la influencia del flujo y de la configuración geométrica de estos equipos.

Sin embargo, los primeros modelos presentados para predecir la eficiencia de estos equipos fueron basados en simplificaciones que no incluían todos los parámetros que afectan su funcionamiento.

De igual manera, las metodologías de diseño de estos separadores estaban principalmente basadas en relaciones empíricas de las diferentes dimensiones del ciclón con respecto al diámetro del mismo.

Por lo anterior, el desarrollo de un modelo matemático para la evaluación del funcionamiento de los ciclones y el análisis del efecto que algunos parámetros del funcionamiento de
los mismos tienen en su eficiencia son presentados en este trabajo.

4. Desarrollo

4.1 Geometría y parámetros de evaluación de los ciclones

Aunque los ciclones pueden tener la entrada de gas en forma tangencial o radial, los de entrada tangencial son utilizados con más frecuencia. Este tipo de ciclón consta de un cuerpo cilíndrico vertical, una sección cónica inferior, una entrada tangencial y una salida superior compuesta por un tubo que penetra en el cuerpo cilíndrico. La salida de las partículas sólidas se hace por el fondo cónico. La porción del tubo de salida que se encuentra en el interior del cuerpo cilíndrico, se conoce como «buscador de remolinos» o «buscador de vórtices», el cual evita que la corriente de gas de alimentación entre directamente a la salida[5]. En la figura 1, se muestra el diagrama de un ciclón típico con entrada tangencial.

Para el diseño de estos equipos, diferentes configuraciones geométricas han sido propuestas. En todos los casos, se toma como longitud característica del equipo al diámetro del cuerpo cilíndrico principal y el resto de las dimensiones se obtiene a partir de relaciones empíricas o semie mpíricas de las dimensiones restantes del ciclón con respecto al diámetro del mismo (D). En la tabla 1 se muestran algunas de las configuraciones geométricas utilizadas para el diseño de los ciclones.

<table>
<thead>
<tr>
<th>Dimensión</th>
<th>Descripción</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Altura de la entrada de gas</td>
</tr>
<tr>
<td>b</td>
<td>Ancho de la entrada de gas</td>
</tr>
<tr>
<td>S</td>
<td>Longitud del tubo de salida</td>
</tr>
<tr>
<td>Dc</td>
<td>Diámetro del tubo de salida</td>
</tr>
<tr>
<td>h</td>
<td>Altura del cuerpo cilíndrico</td>
</tr>
<tr>
<td>H</td>
<td>Altura total del ciclón</td>
</tr>
<tr>
<td>B</td>
<td>Diámetro de salida para las partículas sólidas</td>
</tr>
</tbody>
</table>

Tabla 1 [6].

<table>
<thead>
<tr>
<th>Relación</th>
<th>Lapple (1939)</th>
<th>Stairmand (1951)</th>
<th>Swift (1969)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K = a/Dc</td>
<td>0.5</td>
<td>0.5</td>
<td>0.44</td>
</tr>
<tr>
<td>K = b/Dc</td>
<td>0.25</td>
<td>0.2</td>
<td>0.21</td>
</tr>
<tr>
<td>S/Dc</td>
<td>0.625</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Dc</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>b/Dc</td>
<td>2.0</td>
<td>1.5</td>
<td>1.4</td>
</tr>
<tr>
<td>H/Dc</td>
<td>4.0</td>
<td>4.0</td>
<td>3.9</td>
</tr>
<tr>
<td>B/Dc</td>
<td>0.25</td>
<td>0.375</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Los parámetros usados en la evaluación del comportamiento de un ciclón son: diámetro de corte ($D_e$), eficiencia de separación ($\eta_{D_e}$) y caída de presión a través del mismo ($\Delta_p$).

El diámetro de corte se define como el diámetro de partícula para el cual se tiene un 50% de eficiencia de separación[7].

La eficiencia de separación de un ciclón ($\eta_{D_e}$) se define como la fracción del peso o porcentaje de cada tamaño de partícula, que puede ser finalmente capturada por el ciclón [5].

La caída de presión a través del ciclón está compuesta principalmente por tres componentes: la caída de presión en el ducto de entrada al dispositivo, la caída de presión dentro del mismo y la caída de presión provocada por el ducto de salida del gas.

4.2 Modelos matemáticos propuestos

Se proponen dos modelos matemáticos (GQJ) para la evaluación del comportamiento de los ciclones, uno para la eficiencia de separación $\eta_{D_e}$ y otro para el diámetro de corte $D_{50}$, mostrados respectivamente en las ecuaciones (1) y (2).

$$\eta_{D_e} = \frac{N \pi (\psi D_p)^2 (\rho_p - \rho) V_e}{9 \mu K_s D_c}$$  \hspace{1cm} (1)

$$D_{50} = \sqrt{\frac{4.5 \mu K_s D_c}{N \pi (\psi)^2 (\rho_p - \rho)}}$$  \hspace{1cm} (2)

Donde $\mu$ es la viscosidad dinámica del gas, $\psi$ es un factor de proporcionalidad propio de la geometría de la partícula, $\rho$ es la densidad del gas, $\rho_p$ es la densidad de las partículas, $V_e$ es la velocidad del gas a la entrada del ciclón y $N$ es el número de vueltas completas que da el gas a través del ciclón en el...
vórtice externo transportando a las partículas con la finalidad de que éstas se impacten contra la pared del ciclón y puede calcularse a partir de la ecuación (3).

\[ N = \frac{H + h}{2a} \]  

(3)

El desarrollo de ambos modelos se basa en la suposición de equilibrio en la dirección radial de la partícula a separar de la corriente de gas dentro del ciclón en un instante dado y considera que las fuerzas más importantes actuando sobre la partícula son la fuerza centrífuga y la fuerza de arrastre [8].

El parámetro \( \psi \) es la relación entre el diámetro equivalente y el diámetro de la partícula. El procedimiento para obtener este diámetro se muestra en continuación [8].

\[ V_{ef} = \frac{\pi}{6} \]  

(4)

Asumiendo que el volumen de la esfera es igual al volumen de la partícula analizada \( V_{ef} = V_p \), entonces

\[ V_p = \frac{\pi}{6} D_{eq}^3 \]  

(5)

Despejando \( D_{eq} \) se tiene

\[ D_{eq} = \left( \frac{6V_p}{\pi} \right)^{\frac{1}{3}} \]  

(6)

Por lo que el cálculo de \( \psi \) sería,

\[ \psi = \frac{D_{eq}}{D_p} = \left( \frac{6V_p}{\pi} \right)^{\frac{1}{3}} \left( \frac{1}{D_p} \right) \]  

(7)

En este caso, \( D_p \) es el diámetro de la partícula (la mayor longitud de la misma, en el caso de partículas de geometría irregular), \( D_{eq} \) es el diámetro equivalente de una esfera con el mismo volumen que el de la partícula analizada, \( V_p \) es el volumen de la partícula y \( V_{ef} \) es el volumen de la esfera.

### 4.2 Efecto de la densidad de la partícula sobre la eficiencia de separación

Para conocer los efectos de los diferentes parámetros en la eficiencia de un ciclón se utilizan las dimensiones de un ciclón con el cual se llevaron a cabo experimentos previamente. Las dimensiones de este equipo se muestran en la tabla 2.

La experimentación se llevó a cabo utilizando partículas de forma irregular (hojuelas) para las cuales se tiene \( \psi = 0.21 \). El gas es aire a temperatura ambiente con una densidad aproximada de 1.22 kg/m³ y una viscosidad dinámica \( \mu = 1.8 \times 10^{-5} \) N·s/m², las partículas tienen una densidad de 2.50 kg/m³ y una velocidad del aire a la entrada del ciclón de 14.9 m/s.

El \( D_{eq} \) medido en el ciclón bajo estas condiciones fue de 10.91 μm y el obtenido utilizando el modelo propuesto para el \( D_{eq} \) fue 11.10 μm, el cual es bastante cercano al experimental con un error de 1.74%. Puesto que el modelo para el cálculo del \( D_{eq} \) fue derivado directamente del modelo de eficiencia de separación \( \eta_{sep} \) [8], este último se utiliza para la evaluación de la influencia de los diferentes parámetros analizados sobre la eficiencia de separación del ciclón estudiado.

Para analizar la influencia de la densidad de la partícula sobre la eficiencia de separación, se evaluó la eficiencia de separación del ciclón antes descrito con las condiciones mencionadas para partículas con \( \psi = 0.21 \) y diámetro de 10 μm variando la densidad del material del cual están hechas las partículas. Los resultados obtenidos se muestran en la figura 2.

![Fig. 2. Variación de la eficiencia de separación con respecto a la densidad de la partícula.](image-url)
Fig. 3. Variación de la eficiencia de separación con respecto a la viscosidad de la partícula.

Fig. 4. Variación de la eficiencia de separación con respecto a la altura del cilindro en el ciclón.

Fig. 5. Variación de la eficiencia de separación con respecto a la altura de la entrada de gas en el ciclón.

ciencia de separación del ciclón manteniendo constantes el resto de las variables que influencian la separación.

4.3 Efecto de la viscosidad del gas sobre la eficiencia de separación

A continuación se analiza la influencia de la viscosidad del gas en la eficiencia de separación. El efecto del cambio en la viscosidad dinámica del gas en la eficiencia de separación del ciclón para las condiciones anteriormente establecidas se presenta en la figura 3.

Al observar la figura 3 se puede notar que el efecto del incremento en la viscosidad del gas es el de una disminución en la eficiencia de separación. Esto está directamente relacionado con la temperatura del gas, por lo que podemos afirmar que un incremento en la temperatura del gas producirá una disminución en la eficiencia de separación del ciclón.

Esto puede explicarse fácilmente relacionando el aumento en la viscosidad del gas con un aumento en el arrastre sobre la partícula. La fuerza de arrastre actúa en la misma dirección pero en sentido contrario a la fuerza centrífuga, por lo que al aumentar la magnitud del arrastre se hace más difícil que la partícula alcance la pared del ciclón y sea capturada.

4.4 Efecto de la altura del cilindro (h) en la eficiencia del ciclón

Para tratar de entender el efecto de las dimensiones geométricas del ciclón sobre la eficiencia de separación se presenta el efecto de la variación de la altura del cilindro sobre la eficiencia del ciclón en la figura 4.

Se puede observar que el incremento en la altura del cilindro nos produce un incremento en la eficiencia de separación del ciclón. Esto concuerda con los resultados obtenidos por Hoffmann et al. en 2001 [9], en los cuales encuentran que el incremento en la altura del cilindro produce un incremento en la eficiencia de separación. Sin embargo, también se reporta haber encontrado un límite para ese incremento después del cual, la eficiencia disminuye en lugar de crecer.

4.5 Efecto de la altura de la entrada del gas (a) en la eficiencia de separación

Finalmente, queriendo evaluar el efecto de otro parámetro geométrico se varía la altura de la entrada del ciclón para observar los efectos de esa variación. En la figura 5 se muestran estos efectos.
Puede observarse que el incremento de la altura de la entrada del gas en el ciclón produce una disminución en la eficiencia de separación del ciclón.

Esto puede explicarse ya que al aumentar la altura de la entrada del gas al ciclón, disminuye la velocidad a la entrada del mismo si se mantiene un caudal constante de gas. Al disminuir la velocidad del gas a la entrada del ciclón disminuirá la velocidad tangencial y, por lo tanto, la fuerza centrífuga dentro del separador, lo cual provoca una disminución de las posibilidades de las partículas de ser proyectadas contra la pared del separador y ser, en consecuencia, capturadas.

5. Conclusiones

El análisis paramétrico realizado de la influencia de cuatro variables que intervienen de manera directa en la eficiencia de separación de los ciclones muestra que tanto las propiedades del gas y las partículas a separar como las dimensiones de los separadores tienen un efecto definido sobre la operación de los mismos.

El modelo utilizado para llevar a cabo la evaluación de la influencia de dichos parámetros fue previamente comparado con resultados experimentales obteniendo buenos resultados en la predicción del comportamiento de estos equipos en procesos industriales.

El aumento en la eficiencia de separación del ciclón al incrementarse la densidad de las partículas a separar es un resultado lógico ya que si se tienen partículas con un volumen muy pequeño, un aumento en la densidad del material significa un aumento en la masa de dichas partículas, lo cual se traduce en un aumento en la fuerza centrífuga con la que serán proyectadas hacia la pared del separador, mejorando la eficiencia de separación.

La influencia de la viscosidad del gas en la eficiencia de separación puede fácilmente extrapolarse a la influencia de la temperatura del mismo en la operación de los ciclones, ya que existe una dependencia directa entre la viscosidad de los gases y su temperatura. Por lo anterior, puede afirmarse que un aumento en la temperatura del gas repercutirá en la eficiencia de separación debido al aumento en la fuerza de arrastre ejercida sobre la partícula.

Al analizar dos dimensiones de la geometría del separador, se puede observar que el incremento en la longitud del cilindro aumenta la eficiencia de separación. El aumento en la longitud del cilindro incrementa la longitud del vórtice exterior y con esto se aumentan las probabilidades de que las partículas que no fueron inmediatamente proyectadas contra la pared del ciclón, sean capturadas en un tiempo posterior.

El cambio en el área de la entrada de gas, en este caso ocasionado por el incremento en la altura de dicha entrada, produce una disminución en la velocidad del gas a la entrada del ciclón (para un caudal de gas constante), lo cual repercute directamente en la fuerza centrífuga ejercida sobre las partículas y por ende en la eficiencia de separación del dispositivo.

6. Referencias


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Gas Turbine Fault Recognition Trustworthiness

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1. Abstract

This paper examines three methods of gas turbine parametric diagnosing. The functioning methods are simulated in the identical conditions of gradually developing faults and random measurement errors. The objectives are to tune the methods, to compare them, and to choose the best one on basis of probabilistic criteria of class correct and incorrect recognition. So, main focus of the paper is a recognition trustworthiness problem. A previous research work in this direction is united with new results and they all together are presented in more systematic form as a common approach. Besides the method comparison and selection, other ways to enhance the trustworthiness are described and the perspectives to realize the methods in real condition monitoring systems are analyzed.

2. Resumen

Este artículo examina tres métodos del diagnóstico paramétrico de turbinas de gas. El funcionamiento de los métodos se simula en condiciones idénticas al desarrollo de fallas y errores aleatorios de medición. Los objetivos son afinar los métodos, compararlos y escoger el mejor con base en criterios probabilísticos del reconocimiento correcto e incorrecto de las clases de fallas. Así, el enfoque principal del artículo es el problema de autenticidad del reconocimiento. Algunos resultados de investigación previa en este campo se unen con nuevos resultados y todos se presentan en forma más sistemática como un enfoque común. Además de la selección del método mejor, se describen otros medios para mejorar la autenticidad y se analizan perspectivas de la realización de los métodos en los sistemas actuales de monitoreo.

Key words: Gas turbine fault classification, thermodynamic model, diagnosis methods, fault recognition trustworthiness

3. Introduction

Knowledge of machines’ health through condition monitoring can allow reducing maintenance cost without risk of failure and give industries significant improvements in efficiency. With a new generation of high temperature and high output gas turbine engines the objectives of attaining a high availability and limiting degradation is of vital importance [1]. That is why advanced condition monitoring systems for critical turbomaschines and auxiliary equipment are designed and maintained in recent decades.

To examine the wide range of common deterioration problems, a comprehensive monitoring system must integrate a variety of approaches. In addition to vibration analysis, there are other technologies to be employed such as gaspath analysis also known as aerothermral [1] or thermodynamic performance analysis. Gaspath analysis of turbomachinery presents advanced calculation techniques used to compute and correlate all performance variables of the gaspath. This technology applied in gas turbine monitoring in order to simulate and detect the failures also provides insight into how efficiently fuel is being utilized and so favours a fuel saving.

Gaspath analysis is a multidiscipline incorporating three interrelated disciplines [2,3]: common engine state monitoring, state prognostics, and concerned in this paper detailed diagnostics (fault localization or identification).
The faults exert influence on measured and registered gaspath variables (pressure, temperatures and consumptions of the gas flow, rotation speeds, fuel consumption, and any others). On the other hand, the influence of operational regime changes is much greater. That is why in the localization algorithms, raw measurement data should be subjected to a complex mathematical treatment to obtain final result identified faults of gas turbine modules (compressors, combustors, turbines). Besides the gaspath faults, control system and measurement system malfunctions can be also detected analyzing the gaspath variables [4].

However, a lot of negative factors, which are explained in more detail below, affect the diagnosing process and make difficulties for the correct detection. So, engine fault detection presents a challenging recognition problem.

To review common works on condition monitoring [1] and fault detection [5] as well as works applied to gas turbines [4,6], it can be stated that a simulation of analyzed systems is an integral part of their diagnostic process. The models fulfill here two general functions. The first one is to give a gas turbine performance baseline in order to calculate differences between it and current measurements. These differences (or residuals) do not practically depend on operational regime variations and so serve as good degradation indices. The second function is related with a fault classification. The models connect module degradation and the residual changes assisting with a fault class’s description.

In the eighties and early nineties, any direct use of complex statistical recognition methods in an on-line capacity was prohibitively expensive in time and computer capacity. It was therefore often decided to simplify diagnostic techniques in order to reduce processing requirements. For instance, Maclsaac and Muir [6] used the method based on fault matrices where every class (fault signature) is presented by residual’s signs only. Other example of a simplified technique can be found in [7]. To recognise the classes the author applies linear and non-linear discriminant analysis but he needs to minimize an axis set of the class’s recognition space to reduce processing requirements. However, our statistical simulations of diagnosing process have shown [8] that the mentioned simplifications cause great recognition errors.

Over the last decade there have been significant advances in instrumentation and computer technology which resulted in more perfect approaches such as [4]. The authors propose some enhanced diagnosing methods based on non-linear gaspath models, statistical neural networks, and probabilistic fault identification that promise high confidence. However, this work as many other lacks for a numerical estimation of method’s effectiveness and any comparison with other known approaches.

As opposed to the mentioned works, this paper is concentrated on a trustworthiness problem. On basis of proposed earlier probabilistic indices [9], three methods are optimized and compared in order to choose the best one and give recommendations for practical use. The method analysis is preceded in the paper by a description of developed and applied models.

### 4. Development

#### 4.1 Models used

First of all the diagnosing process needs a base-line to calculate the residuals [5] which may be presented as relative changes of gaspath variables

\[
\delta Y^* = \frac{Y^* - Y_0(U)}{Y_0(U)}
\]

where \( Y^* \) is measured value, \( Y_0(U) \) - base-line value, and \( U \) is the vector of control variables (for example, fuel consumption and ambient conditions (ambient air pressure and temperature). So, the vectorial base-line function \( Y_0(U) \) may be interpreted as a model of gas turbine normal behavior.

Such a normal state model may be formed by any abstract function as well as a physical model. As an example of an abstract function, the second order four arguments full polynomial

\[
Y_0(U) = c_1 + c_2 U_1 + c_3 U_1 + c_4 U_1 + c_5 U_1 + c_6 U_1 U_2 + c_7 U_1 U_2 + c_8 U_1 U_2 + c_9 U_1 U_2 + c_{10} U_1 U_2 + c_{11} U_1 U_2 + c_{12} U_1^2 + c_{13} U_1^2 + c_{14} U_1^2 + c_{15} U_1^2
\]

is given which is able to describe correctly an engine behaviour [10]. To compute the coefficients \( c_1 - c_{15} \) this model needs registered data inside a wide range of operational conditions.

Non-linear thermodynamic model [3], in which every module is presented by its full manufacture performance map as it is done in [6], demonstrates the option of a physical model. The capacity to reflect the normal behaviour is based on objective physical principles realized. Since the faults affect the module performances involving in the calculations the thermodynamic model has additional capacity to simulate gas turbine degradation.
Fig. 1. Correction factor effects.

To have a possibility to displace the module maps of performances \( \tilde{v} \) (corrected flow parameters or efficiency parameters) for a fault development introducing into the model, the correction factors

\[
\Theta_j = \frac{v_j}{v_{j0}}
\]

are introduced as a relative performances, where \( v_{j0} \) is a nominal value. Fig. 1 gives a schematic representation of correction factor actions.

So, in the thermodynamic model, the gaspath variables relate with the control variables and the correction factors i.e. present a vector function of the view

\[
Y(U, \Theta)
\]

This function is computed as a solution of an algebraic equations system reflecting the conditions of a gas turbine modules combined work. The software consists of approximately 60 subprograms, most of them are universal. Thermodynamic models of more than 20 gas turbine engines of various schemes were elaborated and applied in health monitoring systems [3].

It is known that typical gaspath faults cause relatively small residuals (4-6%). That is why a linearization of the functional dependence \( \hat{Y}(\Theta) \) is possible and the linear model:

\[
\delta Y = H \delta \Theta
\]

is widely used in diagnostics. It connects the small relative changes \( \delta \Theta \) of correction factors with the relative deviations \( \delta Y \) of gaspath variables by the matrix of influence coefficients \( H \). The thermodynamic model software is capable to generate the matrices \( H \) for any operational conditions determined by the vector \( U \).

What is a difference between the simulated deviations \( \delta Y \) and the residuals \( \tilde{Y} \) based on real measurements? Ideally, they should be equal however every vector has its own errors.

As described before, either the non-linear model (4) or the linear one (5) are capable to simulate the fault development and for this reason can be classified as diagnostic models. However, fault modeling accuracy presents a separate problem and this is not an object of current study. In this paper, the hypothesis is accepted that the diagnostic models adequately describe the mechanisms of gaspath deterioration; consequently, the vector \( \delta Y \) does not contain errors. In section 4.6, some arguments are given to support the hypothesis.

As regards the vector \( \delta \tilde{Y} \), its errors occur due to measurement errors in \( \tilde{Y} \) and \( U \) as well as possible inherent inaccuracy of the function \( \tilde{Y}(U) \). It is supposed that a systematic component of these errors does not depend on a deterioration development and a random component is normally distributed due to various factors affecting the accuracy of the residuals. As a consequence, the residuals \( \delta \tilde{Y} \) can be presented as a sum of the deviations \( \delta Y \) and the standard normal distribution vector \( \epsilon \), multiplied by the diagonal matrix \( \Sigma \) of maximal dispersions. In this way,

\[
\delta \tilde{Y} = \delta Y + \Sigma \epsilon
\]

Inside the following approach to gas turbine diagnostics and methods' comparison in basis of trustworthiness criteria, the described models aid to form a gas path fault classification.

4.2 Recognition trustworthiness: common approach

The pattern recognition theory supposes three principle stages of total recognition process: a classification forming, a recognizing itself, and a trustworthiness estimating. These stages are concretized below in application to a gas turbine diagnostics.
4.2.1 Classification

Many gaspath faults are known from scientific literature. For instance, Meher-Homji et al. [11] give an excellent discussion of performance degradation mechanisms. Existent fault variety is too great to distinguish all possible gas turbine degradation states, moreover a maintenance personnel does not need such a detailed diagnosing. That is why the degradation states should be divided into limited number of classes.

There are many difficulties to form a representative classification based on real fault appearances only. The faults appear rarely and their displays depend on a fault severity, engine type and operational conditions. A few degradation modes only, for instance, a compressor contamination of stationary gas turbines and compressor erosion of helicopter engines, can be interpreted as a permanent problem.

As a result, the real classification could be theoretically made up only for a great engine fleet maintained over a long period of time and model-based classifications are widely used in diagnostics [2,6].

In this paper, applying the models (4) and (5) the classification is made up in the multidimensional space of the corrected residuals

$$Z_i = \frac{Y_i - Y_{th,0}(U)}{\sigma_i}, i = 1 - m$$

(7)

and a diagnostic decision about the actual state is taken in the same space. Here $\sigma_i$ is a maximal random error amplitude of the deviation $[Y_i - Y_{th,0}(U)]$ and $m$ is a number of measured variables.

So, the residual vector $Z$ corresponds to the vector $Y$. The vector $Z$ corresponding to the measure $Y$ is formed in the same way.

The hypothesis is accepted that an object (engine) state $D$ can belong to one of $q$ determined beforehand classes

$$D_1, D_2, \ldots, D_q$$

(8)

only, as it is often supposed in the pattern recognition theory. Consequently,

$$\sum_{j=1}^{q} D_j = 1 \quad \text{and} \quad P(D_j / D_i) = 0$$

We consider two types of classes: single and multiple.

The single type class has one independent parameter of fault severity, for example, one correction factor or some correction factors changed proportionally. This type is convenient to describe any known permanent fault of variable severity. In Fig. 2, the class $D_1$ demonstrates this type. The point $O$ corresponds here to an engine normal state. The figure $\Omega_1$ presented by the line $O-L_1$ reflects theoretical changes of the residuals $Z$ while fault severity increasing to a limiting engine state in the point $L_1$. The figure $\Omega_1^*$ presents the residuals $Z$ incorporating their random components induced by measurement errors. To complete class forming, any residuals sample or distribution function inside the region $\Omega_1^*$ is required.

In contrast to the single type class, the multiple type class has more than one independent parameter, for example, some correction factors to be changed independently. This class may be useful to combine some faults when their own displays and descriptions are uncertain. The class $D_2$ in Fig. 2 is formed by independent changes of two correction factors, that is why the region $\Omega_2$ presents a surface and the region $\Omega_2^*$ has more complex form.

4.2.2 Recognition

A nomenclature of possible diagnosis

$$d_1, d_2, \ldots, d_q$$

(9)

corresponds with the accepted classification $D_1, D_2, \ldots, D_q$.

To make a diagnosis $d$ a method dependent criterion

$$R_j = R(Z, D_j)$$

(10)

is introduced as a closeness measure between current residual vector $Z$ (pattern) and every item of the classification (8) and a decision rule.
is established.

4.2.3 Trustworthiness

Various negative factors affect the diagnosing process and the final recognition. So, to ensure the diagnosis $d$ it needs to be accompanied by any trustworthiness assessment. Unfortunately, few probabilistic recognition methods only are capable to compute a confidence parameter for a current diagnostic action. On the other hand, such a particular parameter depending on the current measure $Y$ can not serve as a criterion of average method reliability, engine controllability, and general diagnosing effectiveness.

For this reason mean trustworthiness characteristics are determined for every analyzed method by a statistical testing procedure. This procedure repeats numerous cycles of a method action. In every cycle, the procedure generates random numbers of the current class, fault severity, and measurement errors according chosen distribution laws, then computes actual pattern $\tilde{Z}$, and finally takes a diagnostic decision $d$ corresponding to this vector. A square diagnosis matrix $Dd$ (see Table 1) accumulates diagnosing results according the rule

$$Dd_{ij} = Dd_{ij} + 1, \text{ if } (D = D_i) \wedge (d = d_j)$$

All simulated patterns $\tilde{Z}$ compose a testing sample $\tilde{Z}t$. Its volume $Nt$ corresponds to a total cycle number. After a testing cycle's termination and diagnosis accumulation, the matrix $Dd$ is transformed into a diagnosis probability matrix $Pd$ of the same format by a normalization rule

$$Pd_{ij} = \frac{Dd_{ij}}{\sum_{k=1}^{d_i} Dd_{jk}}$$

The diagonal elements $Pd_i$ forming a probability vector $\vec{P}$ of true diagnosis represent indices of distinguishing possibilities of the classes. Mean number of these elements - scalar $\bar{P}$ - characterizes the total controllability of the engine with its measurement system. No diagonal elements give probabilities of false diagnosis and their great values help to identify the causes of bad class distinguishability. These elements make up probabilities of false diagnosis

$$Pe_j = 1 - Pj \text{ and } \bar{Pe} = 1 - \bar{P}$$

also applied in comparative analysis.

<table>
<thead>
<tr>
<th>Diagnosis</th>
<th>Classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Dd_{11}$</td>
<td>$Dd_{12}$</td>
</tr>
<tr>
<td>$Dd_{21}$</td>
<td>$Dd_{22}$</td>
</tr>
<tr>
<td>$\ldots$</td>
<td>$\ldots$</td>
</tr>
<tr>
<td>$Dd_{m1}$</td>
<td>$Dd_{m2}$</td>
</tr>
</tbody>
</table>

The number $Nc$ that determines computational precision of the described indices is chosen as a result of compromise between a time $T$ to execute the procedure and diagnosing accuracy requirements. In any case, uncertainty in the probabilities should be less than studied effects of changes of the method or diagnosing conditions.

4.2.4 Methods

Three recognition techniques which present different approaches in a recognition theory have been chosen for diagnosing. The first technique is based on the Bayesian approach [12], the second operates with the Euclidian distance to recognize gas turbine fault classes, and the third applies the neural networks which present a fast growing computing technique expanding through many common fields of applications. The techniques have been adapted for the diagnosing and statistically tested by the above procedure. While the testing the settings of these diagnosing methods were adjusted and the methods were compared in equal conditions.

4.2.5 Comparison conditions

Fixed conditions in which the diagnosing methods are simulated and compared are described below.

A. Gas turbine operational conditions determined by the vector $\vec{U}$ are: a maximal gas turbine regime established by the compressor rotation speed variable and standard ambient conditions.

B. Measured parameters structure and accuracy correspond to a gas turbine regular measurement system that includes 6 gaspath parameters to be included in the vector $\vec{Y}$. It is assumed that fluctuations of the residuals (7) mainly induced by measurement errors are normally distributed.

C. Classification parameters. Two classification variants are considered.
Table 2. Trustworthiness indices (case of single class type).

<table>
<thead>
<tr>
<th>$P_d$</th>
<th>0.861</th>
<th>0.759</th>
<th>0.745</th>
<th>0.856</th>
<th>0.828</th>
<th>0.834</th>
<th>0.788</th>
<th>0.818</th>
</tr>
</thead>
<tbody>
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<td>0.361</td>
<td>0.759</td>
<td>0.745</td>
<td>0.856</td>
<td>0.828</td>
<td>0.834</td>
<td>0.788</td>
<td>0.818</td>
<td></td>
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<tr>
<td>0.041</td>
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<td>0.030</td>
<td>0.033</td>
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<td>0.023</td>
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<td></td>
</tr>
<tr>
<td>0.016</td>
<td>0.014</td>
<td>0.010</td>
<td>0.011</td>
<td>0.023</td>
<td>0.019</td>
<td>0.023</td>
<td>0.043</td>
<td></td>
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<tr>
<td>0.006</td>
<td>0.004</td>
<td>0.004</td>
<td>0.008</td>
<td>0.003</td>
<td>0.003</td>
<td>0.004</td>
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<td></td>
</tr>
<tr>
<td>0.004</td>
<td>0.003</td>
<td>0.001</td>
<td>0.001</td>
<td>0.004</td>
<td>0.004</td>
<td>0.004</td>
<td>0.004</td>
<td></td>
</tr>
</tbody>
</table>

\[ P_t = 0.8178 \]

Table 3. Trustworthiness indices (case of multiple class type).

<table>
<thead>
<tr>
<th>$P_d$</th>
<th>0.891</th>
<th>0.784</th>
<th>0.016</th>
<th>0.003</th>
<th>0.042</th>
<th>0.072</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.045</td>
<td>0.038</td>
<td>0.028</td>
<td>0.883</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.014</td>
<td>0.016</td>
<td>0.883</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ P_t = 0.8744 \]

4.3 Method 1: Bayesian recognition

For actual measurement $Y$ and corresponding $Z$, the Bayes formula permits to determine a posteriori probabilities:

\[
P(D_j/Z^t) = \frac{f(Z^t/D_j)P(D_j)}{\sum_{j=1}^{q} f(Z^t/D_j)P(D_j)}
\]

(15)

where

- $P(D_j)$ is a priori probability of the class $D_j$ and
- $f(Z^t/D_j)$ is its pattern density function

Density function assessment is a principle problem of statistics. To simplify it, the function $f(Z^t/D_j)$ was presented by elemental distributions $f(Z^t/D_j)$ and $f(Z^t/Z)$.

\[
f(Z^t/D_j) = \left[ f(Z^t/Z) f(Z/D_j) \right] \Omega_j
\]

(16)

and the following assumptions were taken: 1) adequacy of the linear model (5) applied to simulate faults, 2) uniform distribution $f(Z/D_j)$ of the model values $Z$ with a different fault severity, 3) normal distribution $f(Z^t/Z)$ of residual errors.

Pointed assumptions considerably simplified the calculation of the sought function: for single fault classes, an analytical formula to take the integral (16) has been obtained as well as a simple numerical algorithm for multiple ones.

According to the Bayesian rule the recognition decision $d_i$ is taken when $P(D_i/Z^t)$ is maximal in the set $P(D_j/Z^t)$, $j=1...q$.

D. Testing sample volume. Analyzing precision of the averaged probabilities, the sample volume was established as a function of class number $Nt=1 \times 000 \times q$.

In that way, section 4.2 embraces explanations of the approach involved (formation of the fault classification, fault recognition rule, and diagnosis trustworthiness indices) as well as gives common conditions to compare the methods. So, we have all necessary general information to begin presentation of every method. In the following three sections, the methods and their adjustment are described in more details and some trustworthiness characteristics computed in the described conditions are given.
that corresponds with the general criterion (10) and rule (11) if we put $R(D/Z)$. 

To assess average trustworthiness of this method (method 1), the diagnosing algorithm based on Bayesian recognition has been elaborated and inserted inside the testing procedure described above. The resulting probabilities (see section 4.2.3) corresponding to a single type classification are placed in Table 2 and the same data for a multiple type classification are included in Table 3.

From Table 2 it can be seen that the classes $D_2$ and $D_3$ have the lowest distinguishability according $P_l$ and the elevated magnitudes $P_{d_2}$ and $P_{d_3}$ explain the cause – a great mutual intersection of these classes.

Comparison of Table 2 and Table 3 shows individual (for every class by the vector $P_l$) and total (by $P_l$) trustworthiness growth for the multiple classification that is a result of two opposite tendencies. On one hand, the replacement of a single type by a multiple one generally leads to more close class intersection and lower trustworthiness. On the other hand, the significant reduction of class total quantity (from nine to four) has a contrary influence. In our case, the second tendency dominates. Of course, another diagnosing method will change the presented probabilities and, probably, the conclusions.

This is an advantage of the method 1 that every diagnosis made for actual measurement may be accompanied by a confidence probabilistic estimate and, on average, such estimates will be maximal.

However, the method is not without its difficulties. It seems to be too complicated to restore density functions of a general form in a multi-dimensional recognition space utilizing real measurements (patterns). Therefore, simple type classes based on the linear model and ordinary theoretical distributions may be described only.

That is why a class representation directly by pattern sets is considered too as well as the methods 2 and 3 capable to treat them.

4.4 Method 2: Euclidean distance

Such a simplification as density functions replacement on the pattern sets permits simulating a fault severity growth by the more exact nonlinear thermodynamic model and forming complex multiple classes described by three and more correction factors. Furthermore, this permits forming real data based classes of general type without any model assistance and consequently without negative influence of model proper errors. Fig. 3 demonstrates new class representation; pattern sets of four classes are given here in the three-dimensional space $Z$.

The recognition space $Z$ of residuals can be classified as uniform since all residuals have the same dispersion according to the transformation (7). That is why it will be correctly to introduce a geometrical measure of closeness between the current vector $Z$ and the class $D_i$ to be used as the recognition criterion $R_i$ (10). For concerned method, this measure is based on the Euclidian distance between two points in a multidimensional space.

The computational algorithm has been realized inside the testing procedure and incorporates the following items. Firstly, outside the testing procedure, a reference sample $Z_r$ of the volume $N_r$ incorporating the pattern sets $Z_r$ for all classes is composed. Secondly, inside the testing procedure, the criteria $R_i$ are calculated for actual $Z$ of the testing sample and every $Z_r$ of the reference set. Thirdly, the diagnosing decision $d_i$ is accepted by the general rule (11). The trustworthiness indices (13) and (14) are calculated then according to the general scheme (see section 4.2.3).

To select the best criterion type, the following variants were considered:

- variant 1 - mean inverse distance $M(1/d)$ between a testing sample point and reference points;
- variant 2 - mean inverse quadratic distance $M(1/d^2)$ between the testing point and the reference points;
- variant 3 - mean distance $M(d)$ between the testing point and the reference points;
- variant 4 - distance between the testing point and a reference sample gravity center.

In preliminary statistical testing, the variant 2 ensured the best class distinguishability and has been selected for further use.

Because the reference sample volume $N_r$ influences the computational accuracy and executing time in the same mode as it does $N_t$, the value $N_r = N_t = 1000q$ has been accepted to carry out next calculations.

### 4.5 Method 3: neural networks

Neural networks consist of simple parallel elements called neurons. According to the common scheme of supervised learning networks are trained on the known pairs of input and output (target) vectors. The connections (weights) between the neurons change in such a manner that ensures decreasing a mean difference (error) $e$ between the target and network output. Many such input/target pairs are used to adapt a network to a particular function.

Above an input layer and output layer of neurons a network may incorporate one or more hidden layers of computation nodes when high network flexibility is necessary. To solve difficult pattern recognition problems multilayer feedforward networks or multilayer perceptrons are successfully applied [13] since a back-propagation algorithm had been proposed to train them. So, a back-propagation network promises reliable fault recognition and we also included it in the comparative analysis.

The network realized in the method 3 has the next composition depending on the measurement system and fault classification structures.

The input layer vector includes 6 elements since network inputs are residuals. The output vector presents the concerned classes and therefore incorporates 9 elements for the single type classification and 4 elements for the multiple one. Network complexity and resolution capability are in close relation with hidden layers quantity and their nodes numbers and, to a first approximation, the network has one hidden layer of 12 nodes.

Differentiable layer transfer functions are of sigmoid type. In the hidden layer, a tan-sigmoid function is applied; it varies from -1 to 1 and is typical for internal layers of a back-propagation network. A log-sigmoid function operates in the output layer; it varies from 0 to 1 and is convenient to solve recognition problems.

To put the compared methods under equal conditions the same reference and testing samples as used before are applied now as data sources in network training and verifying processes. Firstly, the training algorithm is performed on the sample $Zr^2$. Secondly, the trained network passes a

### Table 4. Training algorithms. "Cycles" refers to the total cycle number of training process. $Pt$ and $P_t^*$ mean probabilities of truthful diagnosis obtained on the teaching and testing samples respectively.

<table>
<thead>
<tr>
<th>$e$</th>
<th>Algorithm</th>
<th>Cycles</th>
<th>Time, s</th>
<th>$Pt$</th>
<th>$P_t^*$</th>
</tr>
</thead>
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<tr>
<td>0.0300</td>
<td>1</td>
<td>383</td>
<td>110</td>
<td>0.8174</td>
<td>0.8132</td>
</tr>
<tr>
<td>2</td>
<td>65</td>
<td>37</td>
<td>0.8132</td>
<td>0.8090</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>111</td>
<td>70</td>
<td>0.8174</td>
<td>0.8172</td>
<td></td>
</tr>
<tr>
<td>0.0290</td>
<td>1</td>
<td>485</td>
<td>127</td>
<td>0.8211</td>
<td>0.8157</td>
</tr>
<tr>
<td>2</td>
<td>95</td>
<td>44</td>
<td>0.8168</td>
<td>0.8126</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>114</td>
<td>71</td>
<td>0.8213</td>
<td>0.8181</td>
<td></td>
</tr>
<tr>
<td>0.0285</td>
<td>1</td>
<td>598</td>
<td>158</td>
<td>0.8234</td>
<td>0.8173</td>
</tr>
<tr>
<td>2</td>
<td>118</td>
<td>49</td>
<td>0.8196</td>
<td>0.8149</td>
<td></td>
</tr>
<tr>
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<td>122</td>
<td>75</td>
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<tr>
<td>0.0280</td>
<td>1</td>
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<td>201</td>
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<td>0.8197</td>
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<tr>
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<tr>
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<tr>
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<td>70</td>
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<tr>
<td>3</td>
<td>178</td>
<td>100</td>
<td>0.8267</td>
<td>0.8228</td>
<td></td>
</tr>
</tbody>
</table>

### Table 5. False diagnosis probabilities (single type classification).

<table>
<thead>
<tr>
<th>Indices</th>
<th>Methods</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td></td>
<td>0.139</td>
<td>0.266</td>
<td>0.170</td>
</tr>
<tr>
<td>$d_2$</td>
<td></td>
<td>0.241</td>
<td>0.248</td>
<td>0.252</td>
</tr>
<tr>
<td>$d_3$</td>
<td></td>
<td>0.129</td>
<td>0.219</td>
<td>0.137</td>
</tr>
<tr>
<td>$d_4$</td>
<td></td>
<td>0.255</td>
<td>0.390</td>
<td>0.250</td>
</tr>
<tr>
<td>$d_5$</td>
<td></td>
<td>0.144</td>
<td>0.224</td>
<td>0.145</td>
</tr>
<tr>
<td>$d_6$</td>
<td></td>
<td>0.172</td>
<td>0.015</td>
<td>0.170</td>
</tr>
<tr>
<td>$d_7$</td>
<td></td>
<td>0.166</td>
<td>0.212</td>
<td>0.143</td>
</tr>
<tr>
<td>$d_8$</td>
<td></td>
<td>0.212</td>
<td>0.243</td>
<td>0.173</td>
</tr>
<tr>
<td>$d_9$</td>
<td></td>
<td>0.182</td>
<td>0.215</td>
<td>0.160</td>
</tr>
<tr>
<td>$\bar{P}_v$</td>
<td></td>
<td>0.1822</td>
<td>0.2258</td>
<td>0.1772</td>
</tr>
</tbody>
</table>
verification stage to compute the probabilistic trustworthiness indices (see section 4.2.3) corresponding to the samples \( Z^r \) and \( Z^r \).

There are a number of variations on the basic back-propagation training algorithm. In order to choose the best one under concrete conditions of fault recognition, twelve variations were tested under fixed given accuracy \( e \) (mean discrepancy between all targets and network outputs) and compared by execution time. Three more perspective algorithms - variable (adaptive) learning rate algorithm (algorithm 1), resilient back-propagation (algorithm 2), and scaled conjugate gradient algorithm (algorithm 3) - were verified additionally for the different accuracy levels \( e = 0.03 \) - \( 0.0275 \), where the value 0.0275 is close to the final obtainable accuracy. The results given in Table 4 show that the algorithm 1 obviously loses the competition, the algorithm 2 seems to be a little more rapid than the algorithm 3 while the last is more reliable on the testing sample. Taking into account our priority - recognition trustworthiness - the scaled conjugate gradient algorithm is selected for next calculations.

An influence of the hidden layer node number was examined too. Above the chosen number 12, the numbers 8, 16, 20 were also verified for options of the chosen back-propagation and fixed cycle number 200. The reduction of the node number from 12 to 8 has demonstrated visible changes for the worse of the obtainable accuracy \( A_p = 0.00103 \) and the probabilities \( P_t = -0.0068 \) and \( P_l = -0.0068 \). On the other hand, the augmentation to 16 and 20 nodes has not improved the algorithm’s characteristics. That is why the node number 12 remains for further calculations.

### 4.6 Methods comparison

To compare all three methods, the statistical testing of the methods 2 and 3 preliminarily checked and adjusted has been performed under the conditions of the method 1 testing and for the same two classification configurations. The resulting trustworthiness characteristics - the probabilities of false diagnosis (14) - are placed in Table 5 (single type classification) and Table 6 (multiple type classification). For the method 1 these probabilities correspond to the data of Table 2 and Table 3.

Firstly, the methods 2 and 3 are compared. They operate with the same input and output data representation and, from this point of view, do not have any significant advantages/limitations one against the other. So, the trustworthiness level and execution time are only arguments to choose the best method.

The indices presented in Tables 5 and 6 demonstrate significantly lower probabilities of false diagnosing by the method 3. Since calculating accuracy for the probability \( P_e \) works out at 0.01, the effect of trustworthiness enhancement is not a result of statistical simulation errors. The comparative calculations repeated for other gas turbine operating conditions prove the conclusion about superiority of the method 3.

As to the methods 1 and 3, the differences between them in diagnosis trustworthiness are lower. As it can be seen in Tables 5 and 6, these methods differ in the probability by 0.0037 - 0.0050. As the differences are opposite by the sign and smaller then the calculating inaccuracy, the trustworthiness levels of the methods 1 and 3 are considered as equal.

However in contrast to the methods 2 and 3, the method 1 applying the Bayesian approach has an advantage to accompany every diagnosis by its confidence estimation. Therefore the diagnostic method based on the Bayesian approach may be recommended for practical application always when we are able to describe the classes by density functions; for example, in the case of model-based classification concerned in this paper. Otherwise, the techniques like neural networks should be used.

### 4.7 Perspectives of practical application

Above the current work focused on a method comparing, numerous investigations were fulfilled by means of a diagnosing process statistical testing and trustworthiness indices analysis in order to study other factors that also affect the gas diagnosing trustworthiness.

The variety of these factors includes:
- measurement system structure;
- measured parameters accuracy;
- number and structure of gas turbine operational regimes including dynamic regimes;
- classification structure and type;
- joint recognition of gas path faults and measurement system proper defects.

Presented trustworthiness analysis may be classified as model-based since gas turbine models describe a normal behavior and fault influence. That is why a reasonable question appears how we could ensure that the obtained results be correct in practice.

To guarantee the results of model-based calculations the statistical testing is carried out for a wide range of possible diagnosing conditions and the conclusions are generalized. In addition, important results are confirmed on real data, if any. For instance, a practical mode has been proposed [10] to enhance the base-line function \( Y_j(U) \) and reduce the deviation errors \( \sigma_j \) by maintenance data analysis; the method based on Bayesian approach was adapted for recognition of physically simulated gas turbine faults and a real compressor contamination and has demonstrated a satisfactory accuracy of the previous model-based realization of the method.

The idea appears of a combined classification: to start a health monitoring system development and operation with model-based classes and later, along with maintenance information accumulation, to introduce one by one the classes formed by real fault description.

5. Conclusions

In this paper, we discuss a statistical testing of gas turbine diagnosing process in order to determine and elevate recognition trustworthiness indices which are averaged probabilities of true/false diagnosis. A thermodynamic model serves to simulate gas turbine degradation modes and form a faults classification. To conduct the trustworthiness analysis in more general form, two types of gas turbine fault classes called single and multiple are considered.

Three diagnosing methods functioning inside the statistical testing procedure are adjusted, verified, and finally compared. Two of them - the method utilizing Bayesian rule and the method applying neural networks - have demonstrated an equally high trustworthiness level and are recommended for the use in condition monitoring systems.

The presented investigation is a part of total work focused on the trustworthiness growth of gas turbine diagnosing; the other investigation lines including an analysis of maintenance data are noted.

Acknowledgments

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6. References

VLSI Fuzzy Cells

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1. Abstract

This work presents excellent basic cells for the construction of logic Trapezoidal Membership Functions (TMF). The proposed cell is composed of the following: current subtract circuit, multiplier/divider and the S-Z shapes circuit. All the circuits presented in this paper use CMOS 0.18 \textmu m technology and work in current-mode. The efficient performance achieved in this work is demonstrated, first, by the individual simulation of the new cells and, second, by the implementation of a decision making system that uses the Mandani inference method and the new TMF cells as the knowledge base. It is important to mention that maximum and minimum circuits were used for the implementation of the Mandani inference method. The key objective of this work is to demonstrate that the improved cells presented are suitable for the implementation of a decision making system based on a fuzzy logic inference method.

Palabras clave: lógica difusa, funciones de membresía, diseño VLSI, sistemas para toma de decisiones.

2. Resumen (Celdas difusas en VLSI)

Este trabajo presenta el desarrollo de celdas básicas para la construcción de funciones de membresía trapezoidales, TMF. Las celdas TMF se encuentran formadas por un circuito de sustracción de corriente, un multiplicador/divisor, y circuitos de forma S-Z. Todos los circuitos presentados en este artículo usan tecnología CMOS de 0.18 \textmu m y trabajan en modo de corriente. El eficiente desempeño de las celdas presentadas en este trabajo se deriva, primero, por medio del funcionamiento independiente de cada una de las celdas nuevas y, segundo, por medio de la realización de un sistema de toma de decisión que emplea el método de inferencia de Mandani junto con las nuevas celdas como base de conocimiento. Es importante mencionar que circuitos de máximo y mínimo fueron usados para el desarrollo del método de inferencia de Mandani. El objetivo principal de este trabajo es demostrar que las celdas mejoradas son susceptibles para la realización de sistemas de toma de decisiones basados en el método de inferencia de la lógica difusa.

Key words: logic, membership functions, VLSI design, decision making systems.

3. Introduction

Fuzzy logic allows the manipulation of information that handles certain degree of membership in a fuzzy array and permits its representation by means of a characteristic membership function. These functions can be related with others, with the use of linguistic variables, in a fuzzy rule that allows the obtention of a conclusion from vague or uncertain information.

The digital and analog techniques constitute the two existent approaches for the hardware realization of fuzzy systems. The features of these techniques make ones more suitable than the other in specific applications.

Nevertheless, for the realization of an efficient fuzzy system, it is required that both techniques contemplate in its design the available time for the rule processing, the space to be occupied by the system and the power it must consume.

The digital approach has a high degree of programmability, but it requires of a analog-digital and digital-analog converters for the interaction with the physical variables that the system works with, besides this turns the system into an array that occupies a considerably great amount of space.

In the contrary way, the analog arrays count with a higher degree of difficulty to be programmed, but in terms of space occupation they are more effective arrays because of the
reduced number of transistors necessary. Analog systems are preferred for its higher processing velocity and its reduced power consumption. Nevertheless, they present certain disadvantages in comparison with the digital systems, the lack of facility to use CAD (Computer Aided Design) tools for its design, and its mayor sensitivity to noise and distortion.

Digital systems must struggle with a lot of problems to minimize the calculus time and the area consumption [14]. Therefore, investment has been done in the research of methods that allow the solution of these problems, without the necessity of the development of dedicated hardware that needs to be adapted for a specific problem [13]. This diminishes the functionality of the digital systems and highlights the necessity to develop fuzzy systems that operate in analog form.

In the other hand, analogous systems present topologies that in terms of complexity and space result quite efficient, but with a low programmability [8], [9], [10]. There are works that offer more complex systems, that present a higher programmability degree [3],[11], [12]. Nevertheless, many of these designs must be improved in order for them to work in the most efficient way in terms of fuzzy logic; this is the case specifically speaking of the work presented in [3]. The topology presented by Camacho in his work offers a clear advantage over the other proposed designs, and this is the generation of asymmetric and symmetric membership functions. Because of this reason, the work of Camacho is taken as the basis for the development of a fuzzy system that operates in an efficient form using Camacho's TMF circuit for the construction of the system's knowledge base.

In regard to analog fuzzy system design, we considered that for this work the best option was to use circuits that operated in current-mode, since the implementation for the functions of subtractions, addition, multiplication, division, minimum and maximum can be done in a simpler way using current mirrors. To this we must add that the current-mode integrated circuits are less sensitive to temperature, they are robust to technology scaling, they are capable to operate with low voltage feeding and they can interact with various sensors [1].

The parameters that can be programmed in a fuzzy integrated circuit include the number of fuzzy arrays that will cover the antecedent rule; the parameters define it's membership functions, the number of arrays for the consequent rules, and the parameters that highlight the rule base.

The key objective of this work is to take the cells that constitute the units that generate the fuzzy logic membership functions and make the appropriate changes to their structure, so they can operate with the most efficient performance possible in terms of fuzzy logic operation. The improved performance of these cells will be proved via the simulation of a decision making system that uses the Mamdani inference method and the new TMF cells as the knowledge base.

4. Development

4.1 Individual Cells

In this section, we will present the different cells needed for the construction of the TMF cell. We will start with the current subtractor, then we will pass to the description of the multiplier/divider, and finally we will proceed with the cell that allows us the construction of S and Z forms.

A. Current Subtract Circuit

The current subtract circuit proposed in [3] is the one presented in Fig. 1. This circuit is in charge of the subtraction of current $I_1$ from $I_2$. While $I_1 > I_2$, the circuit's output current is the result of the subtraction and when $I_1 \leq I_2$, the output is equal to zero. Equation (1) represents the transfer function of the current subtractor in terms of the dimensions of the transistors it uses.

$$I_{out} = \begin{cases} 
\frac{I_{31}W_{M1}I_1}{I_{31}W_{M2}I_2}, & I_1 > I_2 \\
\frac{I_{36}W_{M3}I_2}{I_{35}W_{M6}I_1}, & I_1 \leq I_2 
\end{cases}$$

If the geometrical relationships of all the transistors are kept unitary, then (1) reformulated as

$$I_{out} = \begin{cases} 
I_1 - I_2, & I_1 > I_2 \\
0, & I_1 \leq I_2 
\end{cases}$$
Table 1. Modified geometrical relationships.

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<tr>
<th>Transistor</th>
<th>Camacho [3]</th>
<th>Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>10μ/10μ</td>
<td>1.8μ/0.36μ</td>
</tr>
<tr>
<td>$M_2$</td>
<td>10μ/10μ</td>
<td>1.8μ/0.36μ</td>
</tr>
<tr>
<td>$M_3$</td>
<td>10μ/10μ</td>
<td>3.6μ/0.36μ</td>
</tr>
<tr>
<td>$M_4$</td>
<td>10μ/10μ</td>
<td>3.6μ/0.36μ</td>
</tr>
<tr>
<td>$M_5$</td>
<td>10μ/10μ</td>
<td>1.8μ/0.36μ</td>
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<td>$M_6$</td>
<td>10μ/10μ</td>
<td>1.8μ/0.36μ</td>
</tr>
<tr>
<td>$M_7$</td>
<td>10μ/10μ</td>
<td>0.36μ/3.6μ</td>
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<tr>
<td>$M_8$</td>
<td>10μ/10μ</td>
<td>0.36μ/3.6μ</td>
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</tbody>
</table>

The mirrors formed by transistors $M_1$, $M_2$, $M_3$, and $M_4$ are in charge of introducing current $I_1$ into node 4. The transistors $M_5$ and $M_6$ extract current $I_2$ from node 4. The result of the subtration is taken from node 4 by the mirror formed by $M_7$ and $M_8$. This mirror is in charge also of preventing the output current from being negative; this is the reason why the output current is equal to zero when $I_1 < I_2$.

The original subtraction circuit presented in [3] had the geometrical relationships for every transistor randomly chosen. This caused a load decompensation between the mirror composed by transistors $M_1$ and $M_2$, and the mirror made by $M_3$ and $M_4$. Table 1 shows a comparison between the original dimensions and the geometrical relationships considered in this work for a more efficient performance.

The performance of the subtractor circuit described with anteriority was tested using the PSPICE circuit simulator. In this case, Fig. 2 shows the circuit’s simulation output using Chamacho’s transistor model AM5 0.8 μm and the geometrical relationships of Table 1 presented in this work. As it can be seen, there is a significant error in the current subtract circuit’s output.

In order to improve the performance of the circuit, we used the proposed geometrical relationships stated in Table 1 and we changed the transistor model to MOSIS 0.18 μm. Fig. 3 depicts the output of the subtraction circuit under the new simulation conditions. After the pertinent modifications we reduced the consumption area, minimized the output error and incremented the dynamic range of the circuit from 0-70 μA ($V_{dd}=5$ V) to 0-190 μA ($V_{dd}=3$ V).

Fig. 2. Simulation output presented in [3].

Fig. 3. Simulation output presented in this work.

Fig. 4. Basic Translinear Cell presented in [4].
B. Multiplier/Divider

The multiplier/divider presented in [3] is based on the Generalized Translinear Principle proposed in [4]. With the help of this principle it is possible to perform the next operation:

\[ I_{\text{out}} = \frac{I_x I_y}{I_w} \left( \left( I_x + I_y \right)^2 - I_x^2 - I_y^2 \right) / 2I_w \]  

(3)

Equation (3) represents the desired output function for the multiplier/divider. Using the generalized translinear principle we were able to perform the wanted output function using a series of operations described next. We applied this principle for the creation of a basic translinear cell used by the multiplier/divider.

The generalized translinear principle is based on Kirchoff’s Voltage Law, which states that the algebraic addition of all the voltages \( V_{GS} \) of a loop formed by MOS transistors must be equal to zero, therefore, considering the characteristic quadratic equation of the MOS transistors, we have that [4]

\[ \sum_{\text{CW}} \frac{I_D}{W} \frac{1}{L} = \sum_{\text{CCW}} \frac{I_D}{W} \frac{1}{L} \]  

(4)

To analyze the basic translinear cell of Fig. 4 we must depart from the function proposed in equation (5).

\[ \sqrt{f} + \sqrt{I_w} = 2\sqrt{f + I_w + I_o} \]  

(5)

This equation is obtained by applying KVL to the loop formed by transistors \( M_{1x}, M_{2x}, M_{1y}, \) and \( M_{y} \) of Fig. 4. It can be appreciated, that it has the form of equation (4). If we raise to the second power both sides of equation (5) and solve for \( f \), we reach the function modeled by equation (6).

\[ f = \frac{(I_{in})^2}{4I_w} \]  

(6)

The circuit of Fig. 4 represents the basic translinear cell. This cell performs the function described by equation (6). In order to obtain a multiplier/divider with an output equal to (3) we used three of these cells. Each of these cells performs the operation of equation (6) with a different input current. In this sense, we considered three input currents: \( I_p, I_o \) and \( I_f \). Using these input currents we obtain equations (7), (8), and (9). These equations represent the outputs of the three basic linear cells used by the multiplier/divider.

\[ f = \frac{(I_p + I_o)^2}{4I_w} \]  

(7)

\[ g = \frac{(I_x)^2}{4I_w} \]  

(8)

\[ h = \frac{(I_y)^2}{4I_w} \]  

(9)

Once that the functions \( f, g, \) and \( h \) were defined in terms of the input currents, it is possible to substitute equations (7), (8), and (9) into equation (3) to obtain the desired output function of the multiplier/divider.
and (9) into equation (10) that becomes the total output of the circuit.

\[ I_{out} = 2(f - g - h) = \frac{I_y l_y}{l_w} \]  

(10)

As shown in Fig. 6, there is a significant error between the expected output, and the real circuit response. It is important to mention, that the topology proposed in [6] is not the only one able to perform multiplication and division operations. Even though the circuits proposed in [17] and [18] are able to perform these operations, the work presented in [6] was preferred over the one proposed in [17], because it does not work in weak inversion. The work presented in [18] was also discarded, since this circuit relies on a higher complexity to perform the needed operations. In this sense, efforts were focused on the improvement of the work presented in [6], so an efficient performance could be obtained from this circuit.

Some modifications to the structure presented in Fig. 5 were necessary. One current mirror was added to the outputs of the basic translinear cells corresponding to functions \( g \) and \( h \). This change served to make load compensation in node 6, but forced the addition of a subtraction cell due to the current direction change that these mirrors introduced. This problem was carried to the output of the current subtract circuit, two more mirrors were necessary to maintain the direction of the output of the circuit as stated in [6].

In addition, it was necessary to change the geometrical relationships of transistors \( M_N, M_N, M_N \) and \( M_N \) for the basic translinear cells to be able to work properly. In this case, the relationship established for the transistors just mentioned was 3.6 \( \mu \) / 0.36 \( \mu \). The dimensions of all the other transistors were kept to 0.36 \( \mu \) / 0.36 \( \mu \), transistor \( M_N \) kept a relationship of 0.72 \( \mu \) / 0.36 \( \mu \) in order to comply with (10).

The image shown in Fig. 8 represents the output obtained from the simulation after the modifications previously mentioned were done. Fig. 8 shows a response that follows closely the expected output in a range of 0 to 140 \( \mu \)A. Hence, the modifications proposed in this work improve considerably the performance of the circuit when it is compared with the results obtained in [3].

C. Circuit for S-Z Shapes.

The circuit presented next is the basic fuzzy cell for the construction of membership functions [3]; this cell delivers in
its outputs $S$ and $Z$ functions depending on the input parameters $I_1$ and $I_2$. For its implementation, we considered the lineal behavior presented by the $S-Z$ forms described in [3], this can be seen in Fig. 9.

The models that describe the $S-Z$ shapes in Fig. 9 are described next according to [3], and defined as shown by equations (11) and (12). These equations define the structure of the circuit depicted in Fig 10.

$$f(x)_{S,SHAPES} = \begin{cases} I_{Amp} - I_{Amp} \left(1 - \frac{I_1 - I_m}{I_2 - I_1}\right), & I_1 < I_m \leq I_2 \\ I_{Amp}, & I_m > I_2 \\ 0, & I_1 > I_m \end{cases}$$

(11)

Functioning errors were found in the structure presented by Fig. 10. The output of the multiplier/divider circuit presented a high error due to a load imbalance in node 15. The same problem was found in node 16. These two nodes were used to make current subtractions in order to comply with (11) and (12). The solution found was to use current subtract subcircuit, instead of doing the subtraction in the node. Fig 11 shows the simulation of the $S-Z$ shapes circuit presented in [3].

Additional changes were necessary to obtain an efficient circuit performance. Besides the structural modifications, some of the geometrical relationships needed to be changed. Table 2 shows a list of the geometrical relationships necessary for the circuit to operate properly. In this case, all the P-type transistors use the same relationship; the modifications were needed mostly in the N-type transistor’s dimensions to adjust the mirror’s gain.

The results of the adjustments made to the circuit proposed in [3] can be seen in Fig. 13. Under the new conditions, the circuit delivers $S$ and $Z$ shapes according to the input parameters, eliminating the error shown in Fig. 11.

D. TMF Circuit

There are various methods for the generation of trapezoidal membership functions. Many authors have reported their
designs in multiple publications; these include [8, 12]. Some of these designs, like [8, 10], present very simple structures that have a very reduced implementation area, nevertheless, the poor programmability offered by these designs, puts them in clear disadvantage against the options proposed in [11] and [12].

The work presented in [11] offers the desired programmability, but its implementation is done with BiCMOS technology. The design proposed in [12] is the most similar to the work presented in [3], with the disadvantage that it is unable to deliver trapezoidal and triangular functions with the same circuit.

Besides, the TMF circuit of Camacho is able to generate asymmetric and symmetric membership functions, the structure of the TMF and the parameters that define it are depicted in Fig. 14.

The versatility and high programmability degree offered by the TMF circuit proposed in [3], gives us enough arguments to continue working with this circuit, with the objective to improve its performance. In this case, the TMF is obtained from the subtraction of a S shape function from a second S shape function.
Table 2. Geometrical relationships for the S-Z shapes circuit.

<table>
<thead>
<tr>
<th>Transistor</th>
<th>Geometrical Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-type</td>
<td>0.72µ/0.72µ</td>
</tr>
<tr>
<td>M₁N, M₂N, M₃N</td>
<td>0.36µ/0.36µ</td>
</tr>
<tr>
<td>M₁N, M₂N, M₄N</td>
<td>3.6µ/0.36µ</td>
</tr>
<tr>
<td>M₅N, M₆N</td>
<td>0.32µ/0.32µ</td>
</tr>
</tbody>
</table>

The input parameters described in Fig. 14 define both S shaped functions. This is illustrated clearly in Fig. 15.

In this sense, the TMF circuit is formed basically by two S-Z shapes subcircuits, in combination with a current subtraction. The mathematical model that describes this behavior is defined as

\[ I_{SW} = I_{Amp} - I_{Amp} \left( 1 - \frac{I_{in} - I_e}{I_b - I_o} \right) - I_{Amp} (1 - \frac{I_{in} - I_o}{I_b - I_e}) \]

(13)

Fig. 16 shows the schematic that represents the TMF circuit. Since the performance of all the cells that compose the TMF circuit were previously improved, we considered that the possible error in the circuit's output should be minimal.

Figure 17 shows the simulation output for the asymmetrical trapezoidal membership function. If the value of current Iₐ is adjusted, the circuit then delivers a symmetrical trapezoidal function, as it is shown in Fig. 18. In the same manner, if the values of Iₐ and Iₐ are equivalent, then the circuit shows a symmetrical triangular function in its output, like the one in Fig. 19. Finally, if the value of Iₐ is adjusted, the TMF cell then delivers the asymmetrical triangular function depicted in Fig. 20. With this, it is possible to observe that the circuit output can be programmed or controlled via the input parameters \(I_a\), \(I_b\), \(I_c\), and \(I_d\). The geometrical relationships used were of 0.36µ/0.36µ for the N-type transistors, and of 0.72µ/0.72µ for the P-type.
type transistors. The performance of the TMF circuit proposed in this work is much more efficient than the one obtained in [3]. The modifications done to this circuit make it a proper option in the implementation of a fuzzy system.

4.2. Maximum and Minimum Detection Circuits

The maximum and minimum detection circuits are necessary for the implementation of the decision making system based on the Mamdani inference method. In this section we will describe the operation procedure of these circuits.

A. Maximum Detection Circuit

Two options were considered for the maximum detection circuit. The first one was proposed in [7], but it was discarded because it only works properly with two inputs. The circuit presented in [5] maintains a stable behavior with many inputs, and is the selected circuit for this work.

The image in Fig. 21 represents the basic cell for the maximum detection circuit. This circuit is based on a very simple operation and the cascade connection of various circuits of this kind composes the maximum detection circuit. The voltage in node 4 is associated directly with the input current In1 and the saturation of M2N is dependent of this voltage. At the same time, the voltage in node 4 is copied to node 3, therefore the saturation of M2N and M2N depends also of this voltage. In summary, the voltage in node 4 controls the saturation of all the transistors in the circuit. The mirror formed by M2N and M2N is used to reflect current In1 into node 4.

A maximum detection circuit with two inputs is presented in Fig. 22. As shown in the image, this circuit is composed of two of the basic cells described in Fig.21. In this case, the voltage in node 4 is associated with the maximum input current flowing
in the circuit; this makes that only one basic cell works in the saturation region, and the rest in the triode region. Hypothetically speaking, if \( I_{s_2} > I_{s_1} \) then the voltage in node 4 is associated with \( I_{s_2} \) and as a result, transistors \( M_N \) and \( M_N \) are in saturation. As a consequence, transistors \( M_N \), \( M_N \) and \( M_N \) are in the triode region, because the voltage in node 4, in this moment, is not related with \( I_{s_2} \). This is how the circuit is able to discriminate the maximum input current from the rest. Transistor \( M_N \), operating as a diode, is added to replace a current source used in [5]. The circuit output is taken from the drain of \( M_N \). The simulation of the circuit presented in Fig. 22 is shown in Fig. 23. This image shows that while \( I_{s_2} > I_{s_1} \), the output is equal to \( I_{s_1} \), when \( I_{s_2} > I_{s_1} \) the situation is reversed, and now the output of the circuit delivers \( I_{s_2} \) as depicted in the graphic.

The minimum detection circuit is obtained complementing the maximum detection circuit. In terms of fuzzy logic, connective circuits are used for the implementation of the MIN connector between the antecedent rules, according to De Morgan's law using the following expression [21]:

\[
\text{Min}(I_{s_1}, I_{s_2}) = \text{Max}(I_{s_1}, I_{s_2}) = I_{s_g} - \text{Max}(I_{s_f} - I_{s_1}, I_{s_f} - I_{s_2})
\]

B. Minimum Detection Circuit

Therefore, the minimum current is obtained as the complement of the maximum of the complements. The minimum detection operator can be implemented by complement subcircuits connected to the n inputs and to the output of the maximum
detection circuit. The complement operation can be easily performed in current-mode applying Kirchoff's current law to node S in Fig. 24. There are two types of complements, positive when the current is entering to S and negative when the current is going out from S. In this case we used a positive complement circuit shown in Fig. 25.

We stated before, that the minimum detection circuit is implemented complementing the inputs and the output of the maximum circuit. This is depicted in the image of Fig. 25. We used three TMF cells as the inputs of the MAX cell in the diagram. As it can be seen, the three inputs and the output of the MAX cell are complemented to form the minimum detection circuit. The parameters that define the three TMF's are shown in Fig. 27. The feeding voltage used in this simulation was $V_{dd} = 5 \text{ V}$.

The simulation of Fig. 28 shows that the proposed complement circuit enables the maximum circuit to perform minimum detection operations. As it can be observed in the image, the circuit is able to determine the minimum of the three TMFs that served as inputs. Nevertheless, a problem was found, the maximum detection circuit presents difficulties when all the basic cells in its structure share the same maximum current. In this case, the output is formed by the sum of all the inputs, resulting in a considerable error. This error is eliminated by adjusting the gain of the mirror used in each basic cell for the maximum detection circuit. Since we used a three input maximum circuit, we adjusted the geometrical relationship of transistor $M_N$ of Fig. 21 to $0.36 \mu \text{A} / \mu \text{A}$ for every basic cell used in the circuit. Under these conditions the circuit operates as shown in Fig. 28.

4.3 Case of Study: Decision Making System

In this section, the cells described in past sections will be taken for the implementation of a fuzzy decision making system. This structure is based on a Mamdani inference method (MIN-MAX Inference). The objective of this section is to demonstrate that the proposed cells operate efficiently as part of a fuzzy inference system, in this case, for the water temperature regulation in a bathroom shower.

A. Mamdani Inference Method

The Mamdani inference method is used commonly for its simplicity and high implementation efficacy, this method is also known as MIN-MAX inference. It uses the MIN t-norm as the implication function and a MAX s-norm as the aggregation operator [1].

---

**Fig. 26.** Circuit used for the simulation of the minimum detection circuit.

**Fig. 27.** Simulation parameters for the input TMFs used in the Fig. 26.

**Fig. 28.** Simulation output for the minimum detection circuit.
Table 3. Input-Output relationship for the decision making system.

<table>
<thead>
<tr>
<th>Water Temperature</th>
<th>Ambient Temperature</th>
<th>Cold Water Tap Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Cold</td>
<td>Cold</td>
<td>Close a lot</td>
</tr>
<tr>
<td>2 Warm</td>
<td>Cold</td>
<td>Close a little</td>
</tr>
<tr>
<td>3 Cold</td>
<td>Warm</td>
<td>Close a little</td>
</tr>
<tr>
<td>4 Warm</td>
<td>Warm</td>
<td>Do nothing</td>
</tr>
<tr>
<td>5 Hot</td>
<td>Cold</td>
<td>Open a little</td>
</tr>
<tr>
<td>6 Cold</td>
<td>Hot</td>
<td>Open a little</td>
</tr>
<tr>
<td>7 Warm</td>
<td>Hot</td>
<td>Open a little</td>
</tr>
<tr>
<td>8 Hot</td>
<td>Hot</td>
<td>Open a lot</td>
</tr>
<tr>
<td>9 Hot</td>
<td>Warm</td>
<td>Open a lot</td>
</tr>
</tbody>
</table>

This method is an inference mechanism based on base rules of the form:

Rule 1: if \( x_1 \) is \( A_1^1 \) and \( x_2 \) is \( A_1^1 \) then \( y \) is \( B^1 \)

Rule 2: if \( x_1 \) is \( A_1^2 \) and \( x_2 \) is \( A_2^2 \) then \( y \) is \( B^2 \)

The inferred conclusion, in the form of a membership function, after the application of the base rules is given by [1]:

\[
\mu_y(y) = \max(\min(\alpha', \mu_{A_1}(y)), \ldots, \mu_{A_n}(y))
\]

Where the activation degree of the \( r \)-rule is:

\[
\alpha' = \min(\mu_{A_1}(x_{a_1}), \ldots, \mu_{A_n}(x_{a_n}))
\]


The decision making system designed to prove the operation of these cells, pretends to maintain warm the temperature of the water in the shower, regardless of the input state and

---

**Fig 29.** Membership functions for (a) Temperature in Shower (b) Ambient Temperature and (c) Output.
supposing that hot water valve is always opened and with constant temperature. In this system, constructed with linguistic sentences, it was established that for the water temperature regulation in a daily bath, a person considers that the input signals are the temperature of the water in the shower and the ambient temperature. Taking in count this input variables, a person takes control actions modifying the position of the cold water tap.

Table 3 shows the series of base rules, and the established relationships between the input signals and the expected output signals. Fig. 29 shows graphically the variable ranges

**Fig. 30. Decision making system block diagram.**

**Fig. 31. Stage representation of the decision making system.**
image, the operations start with the membership function generation for the system inputs. The input correspondent to the shower is a current sweep that goes from 0 to 30 μA. In this case, only the TMF output representing «cold» is considered, this can be seen in Fig. 29a. In the other hand, the input correspondent to the ambient temperature was maintained constant with a value of 15 μA, representing «warm». At the same time, the «warm/cold» input condition is being evaluated. These two conditions «cold/warm» and «warm/cold» give the same answer.

The relationship between antecedents is established via the minimum detection circuit to obtain partial answers that are aggregated in the maximum detection stage. In this way, the output of the minimum detection circuit that corresponds to the input with the highest value will be the output of the entire system. This is shown in Fig. 31, where all the operations are performed with the required efficiency. It is important to add, that the addition of a current mirror at the TMF cells output was necessary to couple this stage with the minimum detection circuits.

To prove the operation decision making system, we tested one by one, the series of base rules defined in Table 3, but for practical reasons we only include the simulation of one example in this work. For every case, the system delivered the expected output for a given input combination. With this efficient circuit operation, we conclude that the modifications made to the circuits described in past sections, make the proposed cells suitable for their utilization in more complex systems.

5. Conclusions

This work presents a significant improvement in the performance of the cells initially proposed by Camacho in [3]. In first instance, the technology used was scaled from 0.8μm AMS to 0.18μm. This change represents a great improvement in terms of circuit area utilization; it allows the utilization of the proposed cells in systems that require a minimal circuit layout area. In the same way, the errors that caused an inefficient circuit operation, were identified and corrected. All topologies presented an improved performance after the proposed modifications, in some cases the error was reduced to noise levels. It becomes obvious, that the inefficient operation of one cell is translated into a functioning error in the consequent cell, affecting the entire system performance. This exposes the existent dependency between the system cells.

Table 4 presents a performance comparison of the circuits used in this work under different feeding conditions. In most cases the dynamic range of the proposed cells doubles the one obtained in [3]. At the same time, the power consumption

<table>
<thead>
<tr>
<th>P-type MOSFET</th>
<th>Feeding voltage</th>
<th>Dynamic range</th>
<th>Power consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1 V</td>
<td>0 - 20 μA</td>
<td>0.0794 mW</td>
</tr>
<tr>
<td>3 V</td>
<td>0 - 190 μA</td>
<td>0.240 mW</td>
<td></td>
</tr>
<tr>
<td>5 V</td>
<td>0 - 400 μA</td>
<td>0.4 mW</td>
<td></td>
</tr>
<tr>
<td>1 V</td>
<td>0 - 30 μA</td>
<td>0.121 mW</td>
<td></td>
</tr>
<tr>
<td>5 V</td>
<td>0 - 140 μA</td>
<td>0.364 mW</td>
<td></td>
</tr>
<tr>
<td>2 V</td>
<td>0 - 240 μA</td>
<td>0.66 mW</td>
<td></td>
</tr>
<tr>
<td>1 V</td>
<td>0 - 250 μA</td>
<td>9 e-5 W</td>
<td></td>
</tr>
<tr>
<td>2 V</td>
<td>0 - 2.2 μA</td>
<td>5.43 mW</td>
<td></td>
</tr>
<tr>
<td>5 V</td>
<td>0 - 6.2 μA</td>
<td>40.1 mW</td>
<td></td>
</tr>
<tr>
<td>1 V</td>
<td>0 - 250 μA</td>
<td>7.22 e-5 mW</td>
<td></td>
</tr>
<tr>
<td>3 V</td>
<td>0 - 2.2 μA</td>
<td>17.1 mW</td>
<td></td>
</tr>
<tr>
<td>5 V</td>
<td>0 - 6.2 μA</td>
<td>62 mW</td>
<td></td>
</tr>
<tr>
<td>1 V</td>
<td>0 - 20 μA</td>
<td>0.31 mW</td>
<td></td>
</tr>
<tr>
<td>3 V</td>
<td>0 - 130 μA</td>
<td>6.22 mW</td>
<td></td>
</tr>
<tr>
<td>5 V</td>
<td>0 - 260 μA</td>
<td>20.7 mW</td>
<td></td>
</tr>
<tr>
<td>1 V</td>
<td>0 - 50 μA</td>
<td>1.17 mW</td>
<td></td>
</tr>
<tr>
<td>3 V</td>
<td>0 - 50 μA</td>
<td>3.61 mW</td>
<td></td>
</tr>
<tr>
<td>5 V</td>
<td>0 - 60 μA</td>
<td>6.89 mW</td>
<td></td>
</tr>
</tbody>
</table>

considered. The membership functions that represent the knowledge base are established by means of the relationships stated in Table 3.

In this case, the operating range of the system is defined between 0 and 30 μA. All the membership functions that represent a system input must be defined in this range. Fig. 29a and Fig. 29b show the different membership functions that represent the system inputs. There are three conditions to represent the water temperature in the shower, three for the ambient temperature, and departing from these, the five conditions that represent the system outputs are obtained.

Fig. 30 shows the block diagram for the implementation of the base rules listed in Table 3. The combination for the relationships between the two input variables require nine minimum detection circuits, one for each base rule, and one maximum detection circuit per action to perform. We must add that one input dependent outputs do not require a maximum detection circuit.

Every connection point between the TMF cells and the minimum detection blocks, represent an output reflected by a current mirror in order to direct the output signal to the next block. This condition is not applied between the minimum and maximum detection circuits.

Figure 31 shows the operation of the water regulation system in a graphic way. In this example, the operations done correspond to rules two and three of Table 3. As shown in the
was reduced to half of the one registered by Camacho’s work. In the other hand, the maximum and minimum circuits present simple topologies with a stable operation with multiple inputs. These circuits are very important, since the operation of the inference system is based on their efficient performance.

The fuzzy cell proposed in this work present an efficient operation in individual manner, and as part of a complex system. The TMF circuit offers a clear advantage over the other designs, since it is able to generate programmable symmetrical and asymmetrical membership functions. It also shows a greater flexibility by being constituted by independent cells, same that can be replaced with more efficient topologies, as it was done in this work.

The decision making system not only proved the efficient performance of the proposed cells, it also demonstrated the precision with which a fuzzy system is able to obtain conclusions using rules, that are based on linguistic variables that manage uncertain information.

The improvement of the cells proposed in [3], and the use of the maximum and minimum detection circuits allowed the implementation of a fuzzy inference system. It is important to mention, that the decision making system designed in this work is not the only possibility, these groups of circuits permit the creation of a n-rule system with any decision purpose.

Since, this work corresponds only to the fuzzification stage, it is objective of a further work, the creation of a fuzzy VLSI processor that controls the input parameters of the membership functions, which represent the knowledge base, and that also able to manipulate, with the use of analog switches, the rule configuration and the associative fuzzy matrix. Another action line for the future is the study of the proposed topologies with an analysis perspective of geometrical change sensitivities, mismatches, noise, total harmonic distortion, source noise rejection, in other words, the parametric optimization of the designs. The collection of circuits presented in this work, allows the realization of an integrated circuit with specific application in an area of less than 3mm2, including the needed fuzzy processor and the defuzzification stage.

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Finite-Element Calculation of the SSFR of Synchronous Machines

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1. Abstract

This paper uses different/state-of-the-art finite-element techniques to effectively obtain the standstill frequency response of synchronous machines. It addresses the continuous interest to avoid actual tests in large machines as they represent high costs and risks of damage. This work uses Lagrange multipliers to create two different finite-element meshes that are joined at the air gap region, allowing the existence of element nodes in other elements edges and facilitates the simulation of the rotor body movement. This is not usually possible in conventional finite-element codes, where the convenient use of dissimilar mesh densities for the stator and rotor components of the machine is not allowed. The finite-element model developed in this work also takes explicit consideration of the external circuitry connected to the machine through a simultaneous and effective solving of the field and circuit equations. The simulations follow the procedure of the corresponding IEEE standard. The numerical results agree well with an older and much less efficient finite-element approach, but that has been fully validated.

Key words: Synchronous Machines; Finite Element Method; Standstill Frequency Response Test.

2. Resumen (El uso de elementos finitos en el cálculo de la respuesta a la frecuencia de máquinas síncronas)

Este artículo usa técnicas modernas de elementos finitos para obtener eficientemente la respuesta a la frecuencia de máquinas síncronas en reposo. Responde al continuo interés que existe por evitar pruebas experimentales en máquinas de alta potencia, debido a que implican altos costos y riesgo de daños. En este trabajo se utilizan multiplicadores de Lagrange para crear mallas diferentes, que se unen en el entrehierro de la máquina, y permiten la existencia de nodos sobre lados de otros elementos y facilitan la simulación del giro libre del rotor. Esto no es realizable con códigos convencionales de elementos finitos, donde la unión de mallas de diferentes densidades para el rotor y el estator no es una opción. El modelo de elementos finitos desarrollado en este trabajo también toma en consideración la circuitería externa conectada a la máquina a través de una solución simultánea de las ecuaciones de los circuitos y del dominio electromagnético. Las simulaciones siguen fielmente los procedimientos de la norma IEEE. Los resultados concuerdan muy bien con una aproximación de elementos finitos obtenida anteriormente, menos eficiente, pero que sirve como marco de comparación, ya que fue completamente validada.

Palabras clave: máquinas síncronas; método del elemento finito; prueba de respuesta a la frecuencia.

3. Introduction

Synchronous machines [1] represent the main way to produce substantial amounts of electrical energy. This energy is mostly generated in thermoelectric and hydroelectric plants, where turbines are respectively driven by high pressure steam and falling water in order to convert mechanical into electric energy. The working principles of thermoelectric and hydroelectric generators are the same, but their construction differs somehow due to the markedly distinct speeds found in each type of plant [2]. The synchronous machine has been a subject of research for more than a century, because its electromechanic design can be optimized and due to the growing need of interconnecting more power networks together [3]. The seminal work of Blondel [2] and Park [4] established the basis of the Two-Axis Theory [5], which is
routinely used nowadays by power system analysts. This way, it is possible to obtain high-order equivalent circuits [6-8] that can be used with circuit simulators to properly design and operate modern power systems. Roughly speaking, a synchronous machine is constituted of copper conductors and ferromagnetic materials arranged in such a way as to enhance magnetic fields interaction, that produce winding voltages and an electromagnetic torque to achieve energy conversion. The nonlinearity of the stator and rotor cores along with the distributed currents induced in the rotor body under transient conditions pose a problem that is difficult to tackle with simple/traditional equivalent circuits since the rotor electromagnetic dynamics cannot be properly modelled [9]. Also, the mechanical system connected to the machine must be appropriately taken into consideration, making the problem a coupled one, where the mechanical and electromagnetic systems must be solved simultaneously. There is an accepted equivalent circuit structure [6] that has been generalized using $n$ damper branches, such that the rotor electromagnetic effects can be reasonably represented. The main problem associated with equivalent circuits is that the identification of the circuit parameters is a task that requires testing of the machine so that the measured data can be used with identification techniques [8, 10, 11]. One of the tests that is predominantly accepted for parameter identification is the Standstill Frequency Response (SSFR) test [12] since it can provide parameters for both axes of the machine. Moreover, the test is performed with low excitation levels as to prevent machine damage. Nevertheless, it is a costly test as it requires that the machine be disconnected from the power network, that is, it must be out of service during the test. In addition, saturation and contact resistances of rotor circuits are not properly taken into consideration and semi-empirical corrections to the circuit parameters must be used to account for this phenomenon [3]. In spite of all these disadvantages is commonly and popularly used to define parameters because it results in equivalent circuits that produce acceptable results when used in power system simulations.

This work shows how state-of-the-art finite-element techniques can be used to model the SSFR test, avoiding in this way the need of actual experimentation avoiding high costs and the possibility of affecting energy supply to consumers. The finite element method [13] is a powerful tool that can be used to directly model the electromagnetic behaviour of electrical machines using partial differential equations that are derived from Maxwell’s equations [14]. Synchronous machines have peculiarities that cannot be directly handled with standard finite-element approaches. For instance, the allocation of windings and rotor motion can pose a special definition of current distribution that is not usually handled by standard codes. Moreover, the conventional finite-element approach requires definition of currents but electrical machines are fed with voltage sources and circuitry, resulting in the need of developing non-conventional methods. This work combines special techniques to obtain an efficient finite-element model. It also shows the use of the theory and the implementation of methods that address the interconnection of external circuitry with the field model, as well as the problem of rotor movement. A «strong» mathematical coupling of the field and circuit equations is performed, whereas rotor motion is taken into consideration using Lagrange multipliers.

4. The Electromagnetic Equations

Formal derivation of the partial differential equation that must be solved to obtain the SSFR of a synchronous machine is given in the following paragraphs. The electromagnetic behavior of the synchronous machine is completely described by Maxwell’s equations [14]:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{(a)} \quad \nabla \cdot \mathbf{D} = \rho \quad \text{(b)}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \text{(c)} \quad \nabla \cdot \mathbf{B} = 0 \quad \text{(d)}$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields. $\mathbf{D}$ and $\mathbf{B}$ are the electric and magnetic flux densities. $\mathbf{J}$ is the free current density and $\rho$ is the free charge density. These equations are complemented with the following constitutive relations to take into account material properties of the medium being analysed:

$$\mathbf{D} = \varepsilon \mathbf{E} \quad \text{(a)}$$

$$\mathbf{B} = \mu \mathbf{H} \quad \text{(b)}$$

$$\mathbf{J} = \sigma \mathbf{E} \quad \text{(c)}$$

where $\varepsilon$ and $\mu$ are the permittivity and permeability, respectively, and $\sigma$ is the conductivity. It is often convenient to find some simplifications in order to solve these two sets of equations when low-frequency electric–power devices are analyzed. One important simplification is established when the displacement current ($\partial \mathbf{D}/\partial t$) in (1c) can be neglected under low frequency operation [14]. It is assumed here that the power transfer from stator to rotor in rotating electrical machines takes place instantaneously. Accordingly, $\mathbf{J}$, in equation (1c), represents the current densities impressed from external sources and the internally generated eddy current densities. Another significant simplification is achieved when a two-dimensional approximation of the electromagnetic device is possible. It is a good approximation to consider that the magnetic field distribution of synchronous machines is nearly planar. This
simplification implies that the vector current densities are axially directed, that is, they only possess one component, resulting into a two-dimensional approach. Hence, the electromagnetic equations can be recast in a more manageable form by using the magnetic vector potential [13, 14]. The magnetic flux density vector can be expressed as the rotational of a potential vector field because of its divergenceless nature (see equation (1d)): 

$$\mathbf{B} = \nabla \times \mathbf{A}$$ (3)

where A is the magnetic vector potential, which is also axially directed in planar representations. In order to specify completely this vector field, its divergence must also be defined [14]. The divergence of A in two dimensions is automatically enforced to zero (Coulomb gauge [15]):

$$\nabla \cdot \mathbf{A} = \frac{\partial A_z}{\partial z} = 0$$ (4)

since the magnetic vector potential has only one component in the axial direction and it is constant along it. Equations (3) and (1a) can be combined to show that the electric field E is given by:

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$ (5)

where V is the electric scalar potential. It is now clearly seen that equation (5) can be written as a scalar relationship because the electric field has only one component:

$$E_z = -\frac{\partial V}{\partial z} - \frac{\partial A_z}{\partial t}$$ (6)

Thus, equations (1c), (2b) and (3) can be combined to give:

$$\nabla \times \left( \frac{1}{\mu} \nabla \times \mathbf{A} \right) = \mathbf{J}$$ (7)

which can then be rewritten for the two dimensional case as (using (6)):

$$\left( \frac{1}{\mu} \nabla \cdot \nabla \right) A_z = \sigma (\frac{\partial V}{\partial z} + \frac{\partial A_z}{\partial t}) = \frac{J_z}{\sigma} = \sigma E_z$$ (8)

An important property of the electric scalar potential in (6) is obtained by taking the divergence of the axial current density in (8). It can be easily verified that the value of this divergence is zero, since the divergence of the rotational of a vector field is always zero. As a result, the following expression is obtained from the middle term of (8):

$$\frac{\partial}{\partial z} \sigma \frac{\partial A_z}{\partial t} + \frac{\partial}{\partial z} \sigma \frac{\partial V}{\partial z} = 0$$ (9)

The first term on the left hand side of (9) is also zero due to (4). Hence the second term of this equation shows that the first partial derivative of V must be a constant and, therefore, V varies linearly in the axial direction. Thus, it can be stated that $\partial V/\partial z$ is simply the potential difference $(V_1 - V_2)$ measured at the ends of a conducting region divided by the effective conductor length ($L_{\text{eff}}$):

$$\frac{\Delta V}{L_{\text{eff}}} = \frac{\partial V}{\partial z}$$ (10)

It is this potential difference that allows the coupling of the field equations with external electric circuits, as well as, the interconnection between different internal regions of the electromagnetic model.

Equation (8), subject to boundary conditions, represents the solution of the general two-dimensional electromagnetic transient problem. If the electromagnetic system is excited with sinusoidal sources and the material properties ($\mu$ and $\sigma$) have linear characteristics, then (8) can be written in the frequency domain by simple substitution of $\partial/\partial t$ by $j\omega$:

$$\left( \frac{1}{\mu} \nabla \cdot \nabla \right) A_z = -J_z + j\omega \sigma A_z$$ (11)

where

$$J_z = -\frac{\partial \tilde{V}}{\partial z} = \frac{\tilde{V}_1 - \tilde{V}_2}{L_{\text{eff}}} = -\frac{\Delta \tilde{V}}{L_{\text{eff}}}$$ (12)

can be thought of as the current density impressed by external sources to the field and stator windings of the machine. The symbol $\sim$ defines a complex quantity. The second term on the right hand side of (11) represents the eddy currents induced in the rotor body and windings of massive conductors [16]. Integration of the right hand side of (11) over coil regions gives the total current externally fed to the machine windings. If there are no eddy currents induced in the machine windings (filamentary coils [17]), it is possible to write (11) as:

$$\left( \frac{1}{\mu} \nabla \cdot \nabla \right) A_z = -\frac{\dot{N}_z}{A_{\text{coil}}}$$ (13)
This equation results from (11) by making the second term on
the right hand side of (12) equal to zero and by assuming that
the current density \( J_n \) over the coil winding is uniformly
distributed over its cross section. \( A_{\text{area}} \) is the area occupied by
the winding. In any case, the integration of the right hand side
of (11) or (13) gives the total winding current \( I_{\text{tot}} \).

5. Finite-Element Approach

The Finite Element Method is a powerful tool to solve partial
differential equations such as (11). In this work, a two-dimen-
sional finite element model with triangular and quadrilateral
elements was constructed to solve (11) and obtain the SSFR
response of a turbine generator. It is assumed that the
permeability pattern of the machine is known and fixed, so
that the problem is linear. The magnetic vector potential is
approximated in each finite element by:

\[
\tilde{A}_h(x, y) = \sum_{i=1}^{N} N_i \tilde{A}_i
\]

(14)

where \( \tilde{A}_h \) represents the magnetic vector potential at the
element nodes and \( N_i \) are shape functions \([13]\). A hierarchical
scheme is used here, where the second order approximation is
obtained by adding on new second order terms and degrees of
freedom to the first order polynomial expansion \([18]\). This
approximation is similar to adding terms to a Fourier series in
order to improve it. The set of complex finite-element linear
equations, that represent the original problem (11), can be
obtained through the Method of Weighted Residuals \([13-15]\),
where the error \( \epsilon \), produced by assuming a linear variation
in each element of the problem domain, is forced to zero in an
average sense within the solution domain (\( \Omega \)):

\[
\int_{\Omega} N_i(\tilde{\epsilon}) \, d\Omega = \int_{\Omega} N_i \left[ (V - \frac{1}{\mu} \nabla) \tilde{A}_j - j \omega \sigma \tilde{A}_j + \tilde{J}_o \right] = 0
\]

(15)

where \( N_i \) is used again here as a weighting function, leading
to the well known Galerkin approach \([13]\). For a first order
triangular element, equation (15) can be written at node \( i \) as:

\[
\tilde{\epsilon}_i = \sum_{j=1}^{N} \left[ \frac{1}{\mu} \int_{\Omega} N_j \nabla N_i \, d\Omega \right] \tilde{A}_j - j \omega \sigma \int_{\Omega} N_j \nabla \tilde{A}_j \, d\Omega - \sigma \frac{\Delta V}{L_{\text{eff}}} \int_{\Omega} N_j \, d\Omega
\]

(16)

It has been assumed that the permeability \( \mu \) and the current
density \( J_n \) are uniform over each element. The general system
of equations is formed by merging each element contribution
to a particular node, leading to a global system of complex
equations:

\[
\begin{bmatrix} \tilde{K} \end{bmatrix} \begin{bmatrix} \tilde{A} \end{bmatrix} + j \omega \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} \tilde{A} \end{bmatrix} + \begin{bmatrix} Q \end{bmatrix} \left( \frac{\Delta V}{L} \right) = \begin{bmatrix} 0 \end{bmatrix}
\]

(17)

where \( \tilde{K} \) is a matrix that embodies the first and second terms on
the right hand side of (16). \( C \) and \( Q \) are matrices that
result from the second and third terms on the right hand side
of (16). \( \Delta V \) is a vector of potential differences at winding
terminals. An additional equation for each winding can be
generated by integration of the right hand side of (11):

\[
\int_{\Omega} N_i \left[ (V - \frac{1}{\mu} \nabla) \tilde{A}_j - j \omega \sigma \tilde{A}_j + \tilde{J}_o \right] \, d\Omega = -I_{\text{tot}}
\]

(18)

which can be written in finite-element terms (using a unit weight
for the Galerkin formulation \([13]\)), giving a system of equations
when all the windings are taken into consideration:

\[
\begin{bmatrix} j \omega \bar{Q}^T \end{bmatrix} \begin{bmatrix} \tilde{A} \end{bmatrix} + \begin{bmatrix} \bar{S} \end{bmatrix} \left( \Delta V \right) + \begin{bmatrix} I_{\text{tot}} \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}
\]

(19)

where \( \bar{S} \) is a matrix that describes the position of the winding
terminals. Equations (17) and (19) can be combined, giving the
following system of equations:

\[
\begin{bmatrix} \tilde{K} & 0 & 0 & 0 \\
0 & \begin{bmatrix} C \end{bmatrix} & \begin{bmatrix} Q \end{bmatrix} & 0 \\
0 & \begin{bmatrix} \bar{S} \end{bmatrix} & 1 & \begin{bmatrix} \bar{A} \end{bmatrix} \\
\begin{bmatrix} \bar{A} \end{bmatrix} & j \omega \begin{bmatrix} \bar{V} \end{bmatrix} & \begin{bmatrix} \bar{A} \end{bmatrix} & 0 \\
\end{bmatrix} \begin{bmatrix} \bar{A} \end{bmatrix} + \begin{bmatrix} \bar{V} \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}
\]

(20)

This system of equations must now be completed with the
external circuit equations.

6. Coupling with External Circuity

Rotating electrical machines are fed through voltage sources
such that winding currents are not known before a finite-
element solution can be obtained. Hence, the third term of
(17) is also unknown and therefore the circuit equations of
the external devices connected to the machine must be
established and solved along with the finite-element system
(17) and (19). There are two basic approaches to achieve this
goal: 1) a «weak» solution of the two sets of equations and
2) a «strong» coupling of the two sets. The weak approach
solves the field equations independently of the circuit ones,
but the solution of the field equations is used as input data of the circuit equations. Then, the solution of the circuit equations is used as input data of the field system. This process is iterative and performed in the time domain [19]. On the other hand, a strong coupling involves the simultaneous solution of the coupled circuit-field problem. This approach implicates an augmented matrix of the resultant system and proper combination of the field and circuit equations. There are several ways to achieve a strong coupling and several examples can be found in the literature [20, 21]. Nevertheless, an efficient way of achieving this goal is given here based on [22]. Consider a RLC parallel circuit between nodes 1 and 2, it is not difficult to show through Kirchhoff’s nodal equations that:

\[
\begin{pmatrix}
\frac{1}{L} & -\frac{1}{L} & 0 \\
-\frac{1}{L} & \frac{1}{L} & 0 \\
0 & 0 & \frac{1}{C}
\end{pmatrix}
\begin{pmatrix}
\frac{dV_1}{dt} \\
\frac{dV_2}{dt} \\
\frac{dV_{10}}{dt}
\end{pmatrix}
+ 
\begin{pmatrix}
\frac{1}{R} & -\frac{1}{R} & -\frac{1}{L} \\
-\frac{1}{R} & \frac{1}{R} & -\frac{1}{L} \\
-1 & -1 & 0
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2 \\
V_{10}
\end{pmatrix}
= 
\begin{pmatrix}
I_i \\
I_{i10} \\
I_{i_{10}}
\end{pmatrix}
\] (21)

where \(V_1\) and \(V_2\) are the potentials at the terminals of the RLC circuit arrangement \(I_i\) and \(I_{i10}\) are the net currents entering nodes 1 and 2, respectively. These two currents may be supplied by an external source to the RLC network \(I_{i_{10}}\) is the capacitor current and \(V_{10}\) is the capacitor initial voltage. Now suppose that these elements are to be connected between terminals 1 and 2 of a finite-element winding. Then, the system of equations (20) is complemented in the following form:

\[
\begin{pmatrix}
[C] & \{Q\} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1/L & -1/L & 0 & 0 \\
0 & 0 & -1/L & 1/L & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
\{A\} \\
\frac{d\vec{V}_{10}}{dt} \\
\frac{d\vec{V}_{10}}{dt} \\
\frac{d\vec{V}_{10}}{dt} \\
\frac{d\vec{V}_{10}}{dt} \\
\frac{d\vec{V}_{10}}{dt}
\end{pmatrix}
= 
\begin{pmatrix}
\Delta\vec{V}_{10} \\
I_{i10} \\
V_{10} \\
I_{i_{10}} \\
V_{10} \\
I_{i_{10}}
\end{pmatrix}
\] + 
\[
\begin{pmatrix}
[\lambda] & \{\lambda\} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1/R & -1/R & -1/R & 0 \\
0 & 0 & -1/R & 1/R & 0 & 0 \\
0 & 0 & 0 & 0 & 1/R & 0 \\
\end{pmatrix}
\begin{pmatrix}
\{A\} \\
\frac{d\vec{V}_{10}}{dt} \\
\frac{d\vec{V}_{10}}{dt} \\
\frac{d\vec{V}_{10}}{dt} \\
\frac{d\vec{V}_{10}}{dt} \\
\frac{d\vec{V}_{10}}{dt}
\end{pmatrix}
= 
\begin{pmatrix}
\Delta\vec{V}_{10} \\
I_{i10} \\
V_{10} \\
I_{i_{10}} \\
V_{10} \\
I_{i_{10}}
\end{pmatrix}
\] (22)

7. Coupling Meshes of Different Densities

Motion is intrinsic to most electromagnetic devices. Motors and generators have a stationary part, called stator, and a rotating structure called rotor. The stator and rotor are separated by an air gap. The rotor speed can be constant or variable. Hence, it is necessary to take into consideration the rotor motion within the finite-element model. A static mesh would be destroyed by the movement of the rotor and methods to avoid this must be devised. One possible method would be to have two finite element meshes, one for the stator and another for the rotor, joined by a slip boundary at the air gap [23]. This boundary is divided into equal intervals such that rotation can be simulated if nodal velocities in the rotor side always coincide peripherally with nodes on the stator side. This means that rotor nodes belonging to the slip boundary are not allowed to fall between sides of elements in the stator side. Thus, with this technique the topology of the stator and rotor meshes is unchanged, but the time step must be adjusted if speed is allowed to vary arbitrarily. Another approach can be formulated using finite elements in all the machine materials but the air gap, where an analytical solution to Laplace’s equation can be found and combined with the finite element part that models the rest of the machine [24]. This permits a smooth motion of the rotor, but this method results in a very non-conventional finite-element approach due to the coupling of the analytical solution with the finite-element discretization.

A method analogous with the «slip boundary» approach is devised by defining a band of elements in the machine air gap. The element band topology is changed as the rotor changes its position. This method is known as the «moving bands» technique [25]. Thus, the rotor and stator meshes remain unaltered but the elements in the air gap are distorted until a re-meshing of the band becomes unavoidable.

The slip boundary method can be elegantly modified to enable boundary nodes on the rotor mesh to fall between sides of elements on the stator mesh, this happening at the sliding interface. The magnetic vector potential of both meshes is then coupled using Lagrange multipliers [26]. This technique offers the advantage of avoiding re-meshing. Moreover, the stator mesh can have a very different element density to the density of the rotor mesh. This approach is used here and can be set in the following way. The solution of the field equation (11) can be obtained through the minimization of the co-energy of the electromagnetic domain, that is, the function that minimises the co-energy functional has the additional property of satisfying (11) [15]. This way of solving partial differential equations is mathematically justified with the calculus of variations [15]. The co-energy functional of a domain can be divided into zones and added together to obtain the total co-energy of the system. For
the rotating machine two sub-domains have been defined: the rotor and stator regions. Both regions are bounded by a band of air and joined together at a circular interface $\Gamma$ within the air gap (see figure 1). Hence the total co-energy can be written as:

$$W(\hat{A}) = W'_r(\hat{A}) + W'_s(\hat{A})$$  \hspace{1cm} (23)

where $W'_r(\hat{A})$ is the co-energy of the rotor domain whereas $W'_s(\hat{A})$ is the co-energy of the stator region. Minimization of this co-energy functional leads to the sought solution. However, this simple approach is not complete since the system is constrained to make the magnetic vector potential continuous at the circular interface:

$$\hat{A}_r - \hat{A}_s = 0 \quad \text{on} \quad \Gamma$$  \hspace{1cm} (24)

where $\hat{A}_r$ and $\hat{A}_s$ are the potentials that must be matched at the interface. A minimization constrained problem can be tackled and solved using Lagrange multipliers [27], which lead to an equivalent minimization problem that can be stated as:

$$F(\hat{A}, \lambda) = W'_r(\hat{A}) + W'_s(\hat{A}) + \int \lambda(\hat{A}_r - \hat{A}_s) d\Gamma$$  \hspace{1cm} (25)

The two first terms on the right hand side of (25) are tackled with standard finite-element techniques that result in a matrix system represented by (17). If there are external devices, they are incorporated using the approach presented in the previous section. The line integral of (25) is performed over the whole circular interface and $\lambda$ is a Lagrange multiplier function that can be put in finite element terms by assuming

$$\lambda = \sum N_{\lambda} \lambda_l$$  \hspace{1cm} (26)

Thus, the Lagrange function has also been discretized using shape functions $N_{\lambda}$. This shape functions can be defined using the already available discretization of the rotor or stator domain. In other words, the element edges and nodes of the chosen domain that face the circular interface are used to define the Lagrange shape functions. In any case, the integral is finally written in finite element terms as:

$$\int \lambda(\hat{A}_r - \hat{A}_s) d\Gamma = \sum_l N_{\lambda l} \lambda_l \left( \sum_r N_r A_r - \sum_s N_s A_s \right)$$  \hspace{1cm} (27)

which must be differentiated with respect to all the potentials in both regions, and also with respect to the Lagrange multipliers $\lambda$. Thus, matrix entries that must be added to (17) are generated.

8. Synchronous Machine Finite-Element Model and Transfer Functions

The design data of a 150 MVA, 13.8 kV, 50 Hz turbine generator was available, so that it was possible to construct a time-harmonic finite-element model and obtain its transfer functions by post-processing of the solution. It is important to mention that this machine was part of Great Britain’s power network. The machine geometry is shown in figure 2, where the circular Lagrange interface can be readily distinguished. This way, it is possible to put the rotor in any position. Particularly in this work the rotor has to be positioned such that the stator field is aligned with the d and q axes. The distribution of the stator phase conductors can be appreciated in figure 2 as well. Figure 3 depicts the finite-element mesh. Two meshes (one for the rotor and one for the stator) were constructed with different densities and coupled at the air gap using Lagrange multipliers as explained in the previous section. Figure 4 shows a close up of the air gap, where the coupling interface and the difference in mesh densities can be seen. A Dirichlet boundary condition [13] was enforced at the outer diameter of the stator. Since the magnetic field is periodic, it is only required to model one half of the full geometry. This periodic condition states that the magnetic vector potential along any radial line must be exactly the negative at an angular displacement of $\pi$, that is:

$$\hat{A}(r, \theta) = -\hat{A}(r, \theta + \pi)$$  \hspace{1cm} (28)

Hence, the radial lines, that serve as boundaries of the rotor and stator geometry (figure 2), are enforced to the periodic boundary condition.
The SSFR test [12] requires current excitation at the stator terminals, so no coupling with external circuits is required. The rotor/field winding must be open or short circuited, depending on the transfer function that is to be determined. In the first case there is no need to consider coupling with external circuity, whereas the short circuit condition demands the series connection of the winding resistance and overhang leakage inductance with the field winding modelled within the finite element model. This is achieved with the methodology given in section 5. A constant permeability value, typical of SSFR testing [28], was used to represent the rotor body. The unsaturated permeability value of the stator normal BH curve was used for the stator core [28]. This allows studying the machine under linear conditions. The armature resistance $R_a$ and field winding resistance $R_f$ are assumed to be known parameters from design data.

The most recent IEEE standard (115-1995 [12]) for performing the SSFR test specifies the connections shown in figures 5 and 6 to obtain the three transfer functions that fully characterize the two port network nature of the $d$ axis and to obtain the transfer function of the $q$ axis. Figure 6 applies to the $q$ axis when the field winding is rotated 90 degrees. The transfer functions to be determined are:

\[ Z_d(s) = \frac{\Delta \phi_d(s)}{\Delta i_d(s)} \text{ (a)} \]

\[ sG(s) = \frac{\Delta i_q(s)}{\Delta \phi_d(s)} \text{ (b)} \]

\[ Z_{q0}(s) = \frac{\Delta \phi_q(s)}{\Delta i_d(s)} \text{ (c)} \]

(29)

where $\Delta \phi_d$ and $\Delta i_d$ are the $d$-axis voltage and current at the machine terminals of the two-axis transformed machine, whereas $i_q$ is the current of the field winding. The $\Delta$ symbol emphasises the fact that the tests is performed with small excitations of the stator currents [12], which in turn leads to small values of the induced voltages and currents of the two port network. $s$ is the Laplace’s operator and takes the form $j \omega$ in the frequency domain. $Z_d$ is the $d$-axis operational impedance. $sG(s)$ is the $d$-axis stator current to field current transfer function. $Z_{q0}$ is the armature to field transfer impedance. The $q$-axis quadrature operational impedance is calculated with (29a) when the subscript $d$ is substituted by $q$. Since the synchronous machine is modelled in the $abc$ reference frame, the transfer functions cannot be evaluated directly and an intermediate impedance and a transfer functions are first calculated. The actual armature impedance that can be calculated from the test measurements is:

\[ Z_{aqd}(s) = \frac{\Delta V_{aqd}(s)}{\Delta i_{aqd}(s)} \]

(30)
such that the $d$-axis operational impedance is given by:

$$Z_d(s) = \frac{1}{2} Z_{arm}(s)$$  \hspace{1cm} (31)

This armature impedance is divided by 2 because two stator windings are used to perform the test. An alternative transfer function that deserves consideration is the $d$-axis operational reactance, which is given by the following expression:

$$X_d(s) = \left( \frac{Z_d(s)}{s} - \frac{R_d}{s} \right) \omega_o$$ \hspace{1cm} (32)

where $\omega_o$ is nominal angular speed of the machine. This reactance function can be more conveniently used in the identification of equivalent circuits instead of the impedance transfer function [29]. The $d$-axis stator current to field current transfer function is calculated with:

$$sG(s) = \frac{\Delta i_{pd}(s)}{\Delta i_d(s)} = \frac{\Delta i_{pd}(s)}{\Delta i_{arm}(s) / \cos 30^\circ}$$ \hspace{1cm} (33)

The $\cos 30^\circ$ takes account of Park's transformation from the abc to the dq frame of reference. Finally, the armature to field transfer impedance is obtained from:

$$Z_{adq}(s) = \frac{\Delta i_{pd}(s)}{\Delta i_d(s)} = \frac{\Delta i_{pd}(s)}{\Delta i_{arm}(s) / \cos 30^\circ}$$ \hspace{1cm} (34)

A convenient operational inductance can be calculated from (34):

$$X_{adq0} = \left( \frac{Z_{adq}(s)}{s} \right) \omega_o$$ \hspace{1cm} (35)

This reactance is called armature to field transfer reactance which also facilitates the identification process of equivalent circuit parameters [29].
9. Results and Validation

The finite-element model gives values of the magnetic vector potential at element nodes, whereas currents and voltages are outputs at nodes of the external circuitry, which have been interconnected with the field equations. These values of currents and voltages are those that would be measured in a real test and they are inputted to the transfer function expressions of the previous section to evaluate them. The SSFR test is simulated using a frequency range that goes from 0.001 to 100 Hz, thus complying with the IEEE standard. Figures 7 to 14 show the magnetic flux density distribution obtained from the finite-element solution at different frequencies for the $d$ and $q$ axes. These figures show how the magnetic field is expelled from the rotor body as the frequency is increased. This phenomenon is explained by the fact that the field and eddy currents are induced in such a way that the stator magnetic field cannot get into the rotor body. This also conforms to electromagnetic theory results that state that the penetration of an alternating magnetic field depends on the frequency and the material conductivity and permeability [14]. An interesting point of the screening effect is that at high frequencies the magnitude of the $d$-axis operational inductance comes very close to the value of the stator leakage inductance, because all the stator magnetic flux is forced to circulate through the slot leakage and air gap paths (see figures 10 and 14).

Figures 15 to 18 show the calculated phase and magnitude curves of the $d$ and $q$ axis transfer functions of the machine. The transfer functions were normalized using the per-unit system of Adkins [5]. These figures also depict the transfer functions obtained in [30], which were calculated using a very different finite-element methodology. The curves labelled as

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Fig. 9. Magnetic flux density distribution ($\omega t = 0^\circ$) at 10 Hz. $d$-axis excited and field winding short circuited.

Fig. 10. Magnetic flux density distribution ($\omega t = 0^\circ$) at 100 Hz. $d$-axis excited and field winding short circuited.

Fig. 11. Magnetic flux density distribution ($\omega t = 0^\circ$) at 0.001 Hz. $q$-axis excited.

Fig. 12. Magnetic flux density distribution ($\omega t = 0^\circ$) at 1 Hz. $q$-axis excited.
Fig. 13. Magnetic flux density distribution (αf = 0°) at 10 Hz. q-axis excited.

Fig. 14. Magnetic flux density distribution (αf = 0°) at 100 Hz. q-axis excited.

Fig. 15. d-axis operational reactance.

Fig. 16. d-axis stator current to field current transfer function.

«old method» refer to these transfer functions. Curves labelled as «new approach» refer to the techniques described in this work. It is important to emphasize that the comparison is shown here since the work of reference [30] was validated and therefore the new results can in turn be validated as well. Nevertheless, it is interesting to notice that the finite-element model of [30] does not model the machine in the abc frame of reference, requiring of non-existent sinusoidally distributed windings. Moreover, special knowledge of the non-conventional two-axis approach of Adkins [5] is required to obtain the machine transfer functions. Actually, the development of conversion formulae was necessary to make a proper comparison with the complying IEEE standard results obtained in this work. It can be seen that both methods give similar results, but some discrepancies can be observed in the magnitudes of the operational reactances. These differences are indeed very remarkable because they make evident that flux-linkage harmonics are not taken into consideration in [30] due to the use of sinusoidally distributed windings. Actual distribution of stator winding is employed here, so that all harmonics produced by the machine geometry are properly considered.

Figures 15 to 18 also show some discrepancies of the phase angles of three transfer functions at high frequencies (bigger than 60 Hz). These differences are also interesting since they state that the skin depth was not correctly modelled in [30]. The reason of this assertion resides in the fact that first order elements and a coarser mesh were used in [30], making the modelling of the skin depth inaccurate. On the other hand, a very detailed/ refined mesh of the rotor body (see figure 3) was constructed here at the rotor outer surface. Moreover, second order hierarchical elements were employed to improve simulation results.

10. Conclusions

The SSFR response of a synchronous generator was obtained with efficient and elegant finite-element techniques. Particularly, a method for the connection of the field and circuit equations was presented. This approach is based on the nodal equations of the external circuits and the total current of the finite-element conductors, such that any circuit and conductor configuration can be analysed. A very clever use of Lagrange multipliers has led to the possibility of coupling
Fig. 17. Armature to field transfer reactance.

meshes of different densities, allowing nodes of elements to fall on sides of other elements. Notice that this is not permitted by the conventional finite-element theory. This way, it was possible to create a very dense mesh at the rotor outer surface, where eddy currents are induced over a small skin depth, while keeping a coarser mesh for the eddy-current free stator domain. The finite-element techniques described in this work can also be used for more general transient simulations. The determination of the SSFR test was carried out following the current IEEE standard, without resorting to non-conventional two-axis approaches or to non-existent sinusoidally distributed windings. The synchronous machine transfer functions obtained in this work were validated using proven results of other work [30], showing numerical superiority due to the use of second order elements and a better mesh.

11. References


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